QUARTERLY JOURNAL
OF THE
ROYAL METEOROLOGICAL SOCIETY

Editor: J. S. SAWYER

Vol. 86 JULY 1960 No. 369

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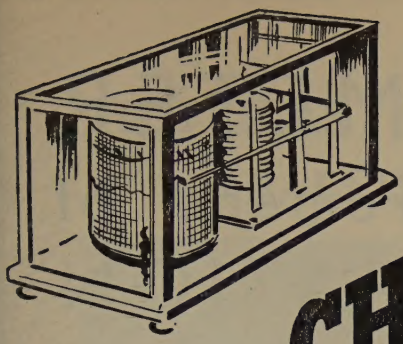

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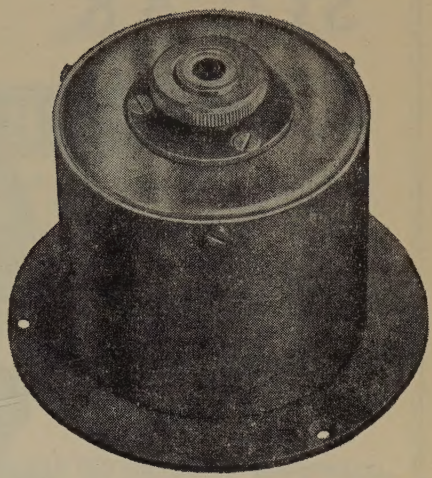

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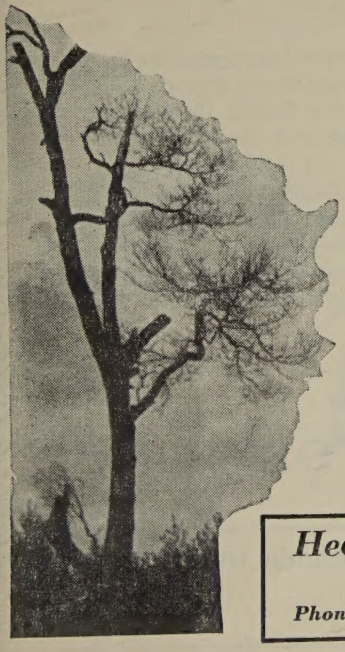

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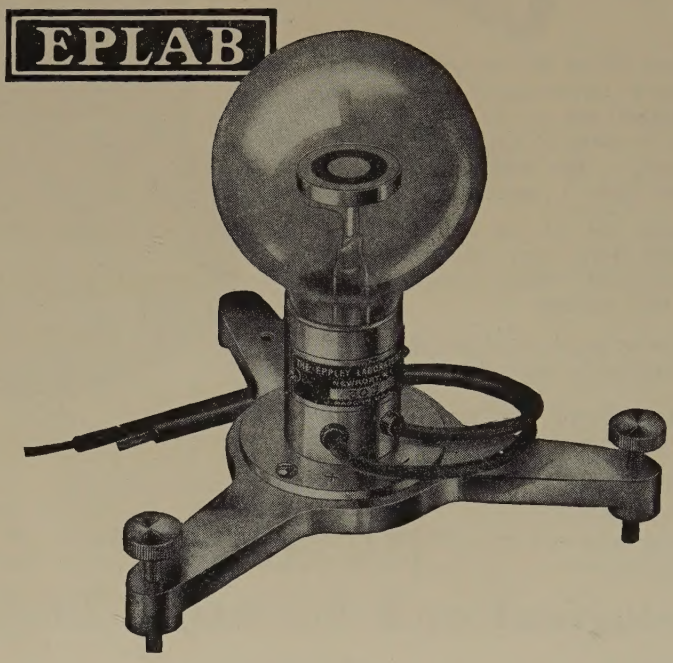

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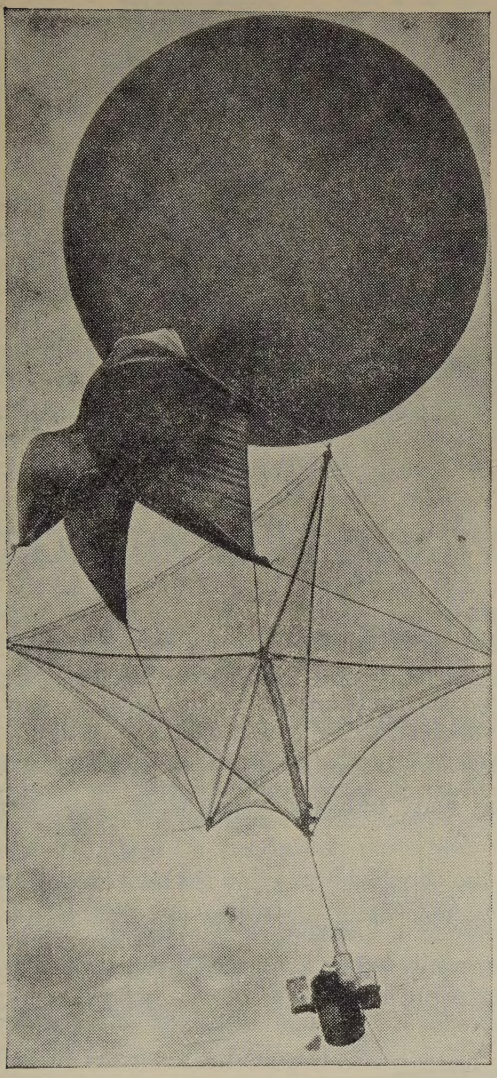

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QUARTERLY JOURNAL
OF THE
ROYAL METEOROLOGICAL SOCIETY

Vol. 86 JULY 1960 No. 369

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Has meteorology become too professionalized ?

By J. M. STAGG, C.B., O.B.E., M.A., D.Sc.

Presidential Address delivered before the Royal Meteorological Society on 20 April 1960

When you honoured me last year by electing me as your President my most serious qualms centred around this address. In trying to allay them I did what I suppose most Presidents do though for other reasons : I browsed over the addresses of my predecessors. That was a stimulating experience : it was both stimulating and enlightening to discover anew the amount of careful thought that lies embedded in the Presidential Addresses through the years. But it was also a little saddening to see how disproportionately little of that thought and advice has gained currency or has even been discussed after the occasion of the address. I believe it might have profit for the Society if the Development Committee put on its agenda from time to time a selection of the suggestions that have been made in the addresses of recent years. I have noted, for example, that each of my two immediate predecessors has suggested, more or less incidentally, that we might do well to take greater interest in what is going on in the higher levels of the atmosphere and in related branches of the physics of the earth. That advice, in my view, was timely if not even overdue, and I would like to keep it alive by taking it as the theme of my address. What any one President may say three times is not necessarily true; but what three consecutive Presidents agree about may be worth looking at a little further. The time is certainly opportune, for as you all know there has been in progress for some time a hardening of claims on particular physical processes and particular regions in the atmosphere. One sign of this hardening has been the renaming and labelling that has been going on, first at the international level and more recently lower down. Now it clearly would not serve our purposes if we imitated this practice by becoming the Royal Society for Meteorology, Aeronomy and Atmospheric Physics, even if our Patron allowed it; but if we are sincere in maintaining that the atmosphere should be treated as a unit we shall have to make up our minds what we are going to do about it.

That it is we, the meteorologists as a body, who have been put in the position of having to re-assert our interests is perhaps not surprising. Between the wars meteorology as a science stood still. Then around 1940 a new era opened up; radio and radar-sounding techniques flooded us with new information, mathematics began to displace description in our literature, and more recently machines have been devised that can cope expeditiously with our most intransigent problems. With all this excitement it is understandable that we should have become engrossed in our own new age almost to the exclusion of what was going on at higher levels and in allied fields. If this seems too harsh and partial a view I can only plead that, rightly or wrongly, it is the view which is widely held by our friends in neighbouring disciplines who have tried to enlist meteorological support in discussions or symposia on the processes and events in those higher levels.

Even as a Society we cannot entirely escape the charge of failing in our responsibility for cultivating our science in its widest sense. For illustration consider the two series of *Reviews* which we have sponsored. In the early 1930's *Problems of Modern Meteorology* dealt with what were then regarded as major problems in particular sectors of the subject : several of them alluded to levels above the stratosphere, and one was wholly devoted to geomagnetic and associated matters. Twenty years later a new series, on broadly similar lines, called *Reviews* was produced, but apart from one, which we had to go outside the

Society to get written, one seeks in vain in the titles of this later series for a suggestion that anything goes on above the troposphere or that any of the neighbouring sciences have interest for the members of our Society.

No one can complain about these things. We have rightly been putting our energies into the solution of those problems of the lower atmosphere for which alone quantitative data have until recently been available. And concentration of effort has certainly paid dividends. We need merely to glance through the *Quarterly Journals* of the last twelve or fifteen years and compare them with the contents of any similar period in the Society's history to realize that within the limits we have set for ourselves we have dug more deeply and put our knowledge on a firmer foundation than at any time in our previous history. This cannot be gainsaid. But the fact remains that in the meantime others have interpreted our concentration on the atmosphere up to 35-40 km as indifference to what is going on elsewhere. And this is not peculiar to our Society : so far as I can make out it is common to similar meteorological bodies everywhere.

As in other spheres at the present time it may be that a wind of change is already beginning to blow in our favour and without any deliberate seeking on our part. For just as radio and radar linked the day-to-day happenings in the lowest layers with those in the stratosphere, so the rocket is beginning to couple the stratosphere to still higher levels. It could be the means of making the atmosphere one whole again if we want it so. Do we or should we want it so, and how high do our interests aspire ? A tentative upper limit which recent literature seems to be feeling towards is 80 km. As all but a minute fraction of what we as meteorologists know at present about the atmosphere is confined to heights below 40 km an additional 40 km should be enough to keep us busy for a number of years. Besides, the particular level of 80 km has more significance than the accident of being about double our present ceiling : it is probably the level of lowest temperature in the whole atmosphere, and from that level upwards dissociation and ionization of oxygen by the sun's short ultra-violet and X-radiation produce conditions of a kind that meteorologists are not supposed to be interested in, or so it could be contended. And anyway, was not the region above 80 km given a separate identity as a specialist's enclave when it was labelled the ionosphere ? It certainly is true that the presence of free electrons and ions introduces complications. But from our viewpoint these electric charges may be treated as little more than a pollution – which incidentally sometimes extends well below 80 km, and the existence of that pollution should not exempt that part of the atmosphere from being a meteorological medium, At 100 km there are still 10^{13} neutral gas particles per cubic centimetre as against 10^5 charged particles, so it is still only a weakly ionized gas with the electrons and ions present only in tracer proportions of about one in a hundred million.

That, as I see it, is the static or compositional state at levels around 100 km and for some tens of kilometers above. Dynamically it is a region of complex and powerful air motion, made up of world-wide and regional circulations and highly localized and turbulent winds. The primary source of energy for these wind systems is heat, as it is throughout the atmosphere. The heat comes mainly from dissociation of oxygen into its atomic form but that does not in any way affect the meteorology. The air motions are still controlled by geostrophic and Coriolis forces and they are free of the immediate boundary conditions we have to contend with.

There are, of course, other kinds of complication in those regions. Movement of the neutral gas cannot take place without starting electric currents, and, conversely, winds of a kind can be generated by interaction between the neutral gas and the charged particles under electric forces : the equations of motion need additional electrostatic and electromagnetic terms. But the point for us is that meteorological principles are involved in this highly dynamic region of the atmosphere : and the question is whether we are content to leave it to other specialists to apply them. My own reading of the literature leads me to believe that if we, the meteorologists, do not interest ourselves in these matters others will take over who are amply capable of assimilating the principles of our science into their own.

Let me not be misunderstood: I am not suggesting that we should devote attention to those higher levels in any vague hope that we shall thereby contribute to the solution of our own conventional problems, least of all that we shall forward our central task, the discovery of the mechanism that controls the large-scale behaviour of the lower atmosphere. I am advocating that we might spare some attention for those higher levels simply because there are meteorological problems there – problems to whose solution we can contribute. And, as a bonus, and in spite of the obvious arguments against the possibility, there is still the outside chance that important functional relationships may lie concealed between large-scale events in the higher regions and those lower down, and we might not like it if others came along and discovered them for us.

Another line of argument for extending our horizons and raising our sights is, I suppose, the obverse of the one I have just been discussing. It is that a restriction of our interests overwhelmingly to the weather atmosphere may have resulted in an unduly inward-looking, practical-result-seeking attitude to our subject. We may have become too professionalized in our outlook, with too little of the imaginative adventurousness that is needed to break old barriers. If that is so it may have reacted adversely on the range of interests represented in our membership and therefore on the opportunities we have for transfusion of ideas. We may have become too insulated from other lines of thought.

It could of course be contended that the complexity of meteorology and the kinds of expertise required even to handle the raw material we have to deal with, leave little scope for what we have regarded as non-essentials: conversely it could be argued that no one without that expertise and who has not grappled with the complexities is ever likely to make a real advance in the subject. I would not be sure of that. In recent years we ourselves have shown how machines can deal fairly effectively with our raw observational data, and how they can then proceed to elaborate a product which, until a few years ago, could have been achieved only by long and skilled experience. Meteorology as we have practised it is no longer a preserve. The future Gauss of the atmosphere may not be a meteorologist at all: he could be someone with his interest transferred from another discipline who can make a fresh and uninhibited approach to the solution of our problems.

It would be ingenuous to think that the process of transfusion of ideas from neighbouring sciences can be easily started. Artificial gimmicks such as the formation of special groups within the Society to study problems common to meteorology and other geophysical subjects would hardly serve. The most direct way is, I suppose, that recently adopted by the Meteorological Office in setting up a branch specificially to study problems of the high atmosphere; that is a welcome sign. There could also be great potentialities, if adopted widely, in the project for geophysical fellowships on the lines of the scheme soon to be started by the Meteorological Office. These fellowships are certainly not being introduced to promote exchange of geophysical ideas among meteorologists; but the holders of the posts can hardly work in a meteorological environment without sharing some of their ideas and absorbing ours. They might become converts to meteorology, or at least take back to their universities strong interests in the subject. The same kind of result could of course be produced if some universities could set up geophysical schools in which teaching and research in meteorology and oceanography, aeronomy and the physics of the ionosphere, geomagnetism and aurorae were carried on under the same roof.

But results from this kind of approach must take time to bear fruit. Meanwhile, for us as a Society, perhaps the best way of achieving our purpose, if we wish it so, is to make a deliberate effort to show that we have interests beyond our bread-and-butter atmosphere. And one practical way of doing this, as Professor Sheppard suggested last year, is to go out of our way to encourage publication and discussion of papers which require us to adjust our perspectives.

Would this extension of interests be a hard uphill business? I think not; for one constant source of surprise in the literature of other geophysical sciences is the frequency with which meteorological concepts and areas of contact appear in unexpected places. Take, for example, the variation in the earth's magnetic field which occurs regularly

each solar day : most of the oscillation takes place in the six hours on either side of local noon : the night hours are flat. A complete theory of this effect has not yet been given but the main principles probably owe as much to meteorology as to electrodynamics. Very broadly they are these. By its gravitational pull the sun produces a tidal oscillation in the earth's air envelope. This pull is exerted primarily on the lower layers because the bulk of the air mass is there, but the isobaric surfaces at every level throughout the column facing the sun are bulged outwards. Near the earth's surface the winds associated with this isobaric configuration are small; but with increasing height they are intensified and altered in phase so that in the layers above 80 km they are increased several hundred-fold and reversed in phase. This comes about only because the two main temperature minima in the atmosphere, at 10-15 km and more especially at 80 km, act as partial traps for the tidal energy of the oscillation and give the column a natural period very near to 12 hours. Added to this tidal effect there is a less clearly understood thermal contribution of about equal importance but of 24-hour period.

At and just below 80 km the winds produced by these combined tidal and thermal effects are probably responsible for the observed direction and speed of noctilucent clouds, visual meteor trails and patches of air glow : in the ionized layers above that level, particularly around 100 km, these wind systems, along with the electrons and ions and the earth's magnetic field, constitute the three essentials of a dynamo. In this comparatively slow but massive machine electromotive forces are generated and these drive the charges into a circulatory current. It is the magnetic field at the ground of these electric currents as they are carried westward round the earth with the oscillation which is recorded as the regular daily variation.

In reality the mechanism is not quite as simple as this : things seldom are. One interesting complication leads to the production of a concentrated ribbon of current in low latitudes, and by analogy with its meteorological counterpart this is called the equatorial electrojet. For our purpose the main point is that in this geomagnetic phenomenon the mechanism is half meteorological. The atmospheric dynamo is a working model of a direct coupling between the lower atmosphere, where the energy both tidal and thermal is fed in, and the higher layers. The winds are straight out of the meteorological textbook, and their intensification and change of phase with increasing height are a direct consequence of the peculiar temperature structure of the atmosphere.

Apart from the mechanism of the thermal contribution, other meteorological aspects of the magnetic daily variation deserve attention. For example, its amplitude is apt to change abruptly from one day to the next. The output of the overhead dynamo can be affected by a patchiness of ionization but it can also be altered by variations in the wind which drives the armature. These wind variations could arise either because the thermal tide makes a varying contribution to the total effect, or because the resonance in the oscillation is liable to be less complete on some sequences of days than on others; and as the resonance is mainly controlled by the temperature minimum at 80 km there might here be an indication of what goes on at this important level. Again, the day-to-day changes in the magnetic variation are small compared with the slow systematic changes from winter to summer, and from the minimum years of each cycle of solar activity to the maximum years. On this time scale the amplitude of the mean daily magnetic variation is a measure of the output of the short wave (910-1,000 Å) solar radiation responsible for the ionization around 100 km. It might seem that this measure should also be used for correlating with regional measurements of total ozone, for the reason that the sun's radiation which is mainly responsible for the ozone (1,760-2,026 Å) is unlikely to vary independently of the output in the 1,000 Å region. And yet the fact is that the magnetic variation (and also the E-layer conductivity measured quite independently) increases by 50 per cent from the minimum to the maximum of the average solar cycle whereas there is no sign of a corresponding change in the production of ozone.

I cannot refrain from mentioning that the Society has a proprietary interest in the atmospheric dynamo, for it was Balfour Stewart who first put forward the hypothesis that the daily magnetic variation might be produced in this way : Schuster and Chapman

transformed the hypothesis into an acceptable theory, and Watson-Watt gave the name to that part of the atmosphere in which the dynamo works. All four were Fellows of this Society and two were my predecessors in this chair.

And now, in contrast with these regular daily variations that are imposed on the earth's main field from outside, let us look very briefly at a much more deeply seated phenomenon, the slow secular change in the main field itself. It is this secular change, you recall, which requires navigators' maps of declination, or variation as they call it, because it is always altering, to be brought up to date every few years. The field as a whole alters slightly but the greatest changes are concentrated in regions which drift slowly westward and alter their size and shape and rate-of-change as they drift.

If it has any analogy in meteorology the problem of accounting for the slow changes in the earth's magnetic field is on a par with explaining those modifications in the circulation of the atmosphere that are of large enough scale to produce variations in climate. As the changes in the magnetic field, like the field itself, can be proved to originate within the earth, it might look as if any similarity ends there, yet the remarkable thing is that some of the main processes that are now believed to produce those magnetic secular changes are just of a kind with those which are associated with the atmosphere's circulation. This comes about because as well as having a fluid atmosphere the earth has a fluid core. In this core adiabatic gradients of temperature, radially outwards toward the solid mantle, set up cells of convective flow. The circulation in the rising and sinking fluid of these convective cells is controlled by Coriolis forces : and since the material of the core is electrically conducting, this circulation in the presence of the basic field constitutes a system of electric currents. Changes in the circulation around the cells can explain the slow changes of magnetic field over the earth as a whole and regionally. In passing I should remind you that these changes leave their imprint on the intensity and direction of magnetization of the rocks that have solidified and the sediments that have been laid down at various stages in the earth's history; this branch of study is called paleomagnetism.

It is perhaps not altogether surprising that this kind of explanation of the slow changes in the earth's magnetic field has a recognizable flavour : it was first suggested by Elsasser after he had made important contributions to meteorology. And there is another interesting link. It was Halley who first formulated the problems of the earth's field and the general circulation of its atmosphere. That was nearly three hundred years ago, and only recently has a beginning been made in the elucidation of either of them. I may also note that whereas the first award of the Napier Shaw prize was for an essay on ' The general circulation of the atmosphere,' the second award was for an essay on ' Climatic change ' and it was won by a specialist in paleomagnetism. It now seems that when we seek insight into the mechanism of the atmosphere's circulation through evidence of its large-scale changes in past ages, one of our trustworthy clues will come from the past history of the earth's magnetic field.

This is not a talk on the physics of the ionosphere or the earth's magnetism. My object in bringing to your notice one or two topics in these subjects is simply to suggest that we do not need to go far in either of these sister sciences before we discover regions of overlapping interest with meteorology either in the physical processes they employ or in the areas where they operate. And this in turn has been part of my theme that meteorology stands to benefit by getting itself a bit more mixed-up than it has been recently with some of those neighbouring sciences.

But one aspect of this mixing needs watching : if it is too superficial it can easily lead along false trails. To explain what I mean I shall sketch very briefly a few of the basic ideas behind disturbance in the earth's magnetic field. Conditions are seldom as quiet as those we were discussing a few minutes ago; the typical record has irregular oscillations and movements superimposed on it. Now it can be shown that these perturbations are produced primarily by electric currents outside the solid earth, some in the ionized layers of the atmosphere and others probably much farther out than that. The only other thing that need be said, or that can be said with confidence, is that by processes

still not clearly defined these currents are to be related to events on the sun. One hypothesis is that streams of corpuscular radiation are shot out from particular regions on the sun. As a stream approaches the outer regions of the earth's magnetic field it reaches a stage when it is diverted around the earth, leaving or replenishing a toroidal current ring at several earth's radii. From this ring, or directly from the stream itself, some of the charged corpuscles are guided by the earth's magnetic field into the outer atmosphere and there produce aurora and great electric currents. It is the effects of the magnetic field of these currents and the current in the ring that are recorded at the surface as magnetic disturbance, or magnetic storms when they are violent.

What, you may ask, has all this to do with meteorology ? On the face of it, very little, except perhaps that the sun is directly or indirectly the ultimate cause of disturbance in the earth's magnetic field as it is of the circulation of its atmosphere. But the surprising thing is that storms in the magnetic field seem to exert a peculiar fascination on meteorologists. In the literature of the last few years I have noted more than a score of papers with titles like ' Geomagnetic storms and changes in the pattern of atmospheric flow at 5 mb.' Most of these articles are in English, but not many, I am glad to say, in *Quarterly Journal* English.

I suppose I should know the reason for the appeal of this part of geomagnetism, for in my earlier days I too succumbed and started one of those cross-bred hares that has not been laid to rest even yet. Perhaps the special temptation to seek relations between magnetic disturbance and the atmosphere is that so little is really known about the connecting links on either side – the sun and the behaviour of the lower atmosphere, and the sun and disturbance. Quite unexpected things may turn up – that is one reason. The other, probably, is the long-established custom for characterizing the degree of disturbance by a numerical index so that it is too easy to segregate the least from the most disturbed days. Those who are inclined that way, therefore, have a ready-made criterion, which they can use for correlating with their own particular indices of the misbehaviour of the lower atmosphere.

Of course it is a legitimate activity to seek relations between the phenomena of different disciplines, and it is true that Nature's concealed mechanisms have occasionally been uncovered by empirical inquiries of this kind. So provided the Editor maintains his high standard of caution in dealing with any cross-relation papers that come his way there is no reason for discouraging this sort of activity. But until a little more is known about the physical processes that are involved it might be more satisfying for all concerned if we gave attention to the somewhat better understood phenomena, a few of which I have tried to indicate to you in this address.

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Total ozone and the middle stratosphere over arctic and sub-arctic areas in winter and spring

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(Manuscript received 30 November 1959)

SUMMARY

It is well known that the arctic stratosphere customarily exhibits a rapid transition from winter to summer which has a permanent warming effect on stratospheric temperatures and causes a marked decrease in the intensity of the circulation at mid-stratospheric levels. Prior to this final warming, temporary warmings occur with a marked effect only in sub-arctic areas. These phenomena are closely related to the rapid increase in total ozone which takes place in the late winter and early spring in polar regions. The major ozone maximum occurs at the time of the final warming, with a magnitude dependent on the intensity of the rapid warming at high levels in the stratosphere. Earlier baroclinic waves during the winter cause closely correlated ozone changes, resulting in a stepwise increase in ozone prior to the final warming.

10-day running means of total ozone and of 100 mb temperature are compared for stations in northern Europe and in Canada; these reveal the relations between long-period changes and processes in total ozone and in the middle stratosphere. In particular, one may thereby explain the seasonal behaviour of total ozone, which departs so markedly from photochemical prediction, as well as the vital dynamical role played by active centres in the stratospheric warm belt (especially in the Aleutian area) in the production of warm ozone-rich stratospheric air masses.

1. INTRODUCTION

In this paper, we shall consider two basic aspects of total ozone behaviour – its seasonal distribution (in particular, the variations with latitude, longitude and year of the relatively rapid rise to the early spring maximum) and its correlation with middle stratosphere temperature. As we shall see, these two aspects are far from being unrelated. In what follows, the terminology of Kochanski (1954) and Hare (1960a) has been adopted with regard to lower and middle stratosphere. The lower stratosphere, from the tropopause to a level of variable pressure in the range 50 to 100 mb, is characterized by warm troughs

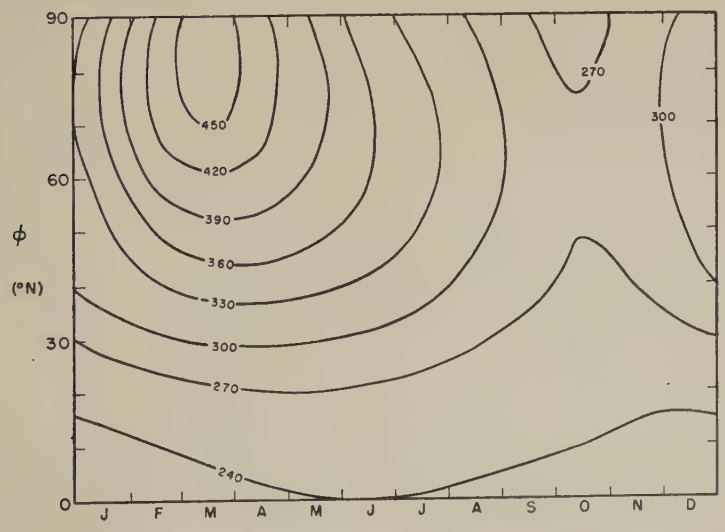

Figure 1. Mean distribution of total ozone as a function of month and latitude.

and cold ridges and by vertical motions generally of the same sign as those in the upper troposphere. The middle stratosphere, which extends to above the 10 mb level, is characterized by cold troughs and warm ridges and by vertical motions generally of the opposite sign to those in the lower stratosphere.

The average variation of total ozone with month and latitude is shown in Fig. 1, adapted from Normand (1953). The isopleths give total ozone in 10^{-3} atm-cm (STP cm), on the Vigroux scale. Total ozone is a minimum at the equator, and a maximum at very high latitudes in late winter and in spring and at moderately high latitudes in the autumn months. Except in the tropics, there is a minimum of total ozone near October and a maximum in the period March to April. The data presented in this figure represent averages over many years and many longitudes. Monthly-mean ozone amounts can vary appreciably from year to year (Perl and Dütsch 1959; Langlo 1952), particularly in the late winter and in spring. It will be noted that, on the average, January and February represent the period of greatest rate-of-change of mean ozone at high latitudes. This period is also variable in timing and in intensity from year to year (Langlo 1952).

Since ozone is created in the atmosphere almost exclusively by photochemical processes, one might expect that photochemical theory would account for most of the features seen in Fig. 1. The computations involved are rather complex, but various authors are at least in qualitative agreement on the theoretical predictions (here we shall employ results given by Dütsch (1953a and b)). The chief results of such calculations are: total ozone should have a maximum in June and a minimum in December, and a maximum at the equator (in all seasons) and decrease towards the poles. These two major predictions are almost completely wrong. Numerically, the total amounts of ozone are predicted to be far too small (except near the equator), although the maximum concentration (ozone density) is correctly estimated to lie between 20 and 30 km.

These photochemical calculations are based on the assumption of equilibrium concentrations, but unfortunately this concept is valid only at levels well above the maximum in ozone density. Thus, for example, the times of half-restoration of equilibrium concentration (after complete destruction at any level) at 45° latitude are: at 30 km, 10 days in summer and $2\frac{1}{2}$ months in winter, and at 35 km, 2 days in summer and 20 days in winter. In the middle and lower stratosphere, solar control of ozone amounts will be negligible compared to dynamic processes, and even on a seasonal basis meteorological effects may well predominate over solar effects. This latter hypothesis may, indeed, be tested by computations of non-equilibrium ozone amounts (steady only with respect to the year as a unit), but these do not greatly improve the theoretical predictions. At 45°N, the total ozone should still be a maximum in June and a minimum in December; at 70°N, the theory is even less realistic, predicting a minimum of total ozone in February and a maximum in June.

Since the rapid increase of total ozone to an early spring maximum has not a photochemical origin, it must be caused dynamically, by processes in the lower and middle stratosphere, where ozone amounts and variability are both high. Most writers on this subject (specific references will be given later) agree that, in some way, this rapid ozone increase is related to the strong meridional temperature gradients in the arctic and subarctic stratosphere in mid-winter, and to the arctic stratospheric jet stream (Lee and Godson 1957; Godson and Lee 1958; Pogosyan 1959). Normand (1954) believed that the low stratospheric temperatures (20 to 30 km) in polar regions from December to February might provide the key to the rapid ozone increase, since stations nearer the pole have more asymmetric annual variations in total ozone, with briefer and earlier, yet more intense, maxima. Schröer (1949) and Moser (1949) assigned the major role to vertical eddy diffusion, enhanced by stronger shears and lesser stability in the vicinity of the arctic stratospheric jet stream. On the other hand, Paetzold (1956) and Dütsch (1956) favoured vertical fluxes on the synoptic scale or mean meridional circulations (in vertical planes), in part since there seemed to be no major source region for the horizontal advection of ozone-rich stratospheric air masses during the period of rapid ozone increase.

2. Behaviour of arctic winter stratosphere

Recent studies of the arctic winter stratosphere have shed considerable new light on the problems mentioned and the present study was, in fact, undertaken to see if the new evidence could supply the required explanation for the rapid ozone increase. At the risk of flagrant over-simplification, we can describe the temperature field at high levels in the arctic stratosphere in the following manner (see, for example, Warnecke 1956; Godson and Lee 1958; Wexler and Moreland 1958; Teweles 1958; Zubyan 1959; Hare 1960a and b). During summer months, the arctic stratosphere is very warm with very small temperature gradients either in the vertical or in the horizontal. During autumn months, relatively rapid cooling sets in, with an intensity that increases with both elevation and latitude. Minimum temperatures in the far north (e.g., 80°N) are usually achieved in January, followed by a slow rise and occasionally a pronounced secondary minimum in late February or early March. Further south (e.g., 70-75°N) a similar behaviour is experienced except that long-period temperature waves (periods of the order of 15-30 days) are superimposed on the much smoother variations at 80°N. As one progresses southwards, these baroclinic temperature waves increase in amplitude to a maximum which is usually near 60-65°N, although there are winters (especially in North America) when the sub-arctic remains mild, with only weak incursions southward of cold stratospheric air. This is particularly true of the Alaska-Aleutian area which is generally characterized by an extremely warm ridge at high levels through most of the winter. In general, these are of very long wavelength (wave numbers of one to three) and are virtually uncorrelated with tropospheric wave trains. The amplitudes of these baroclinic temperature waves change markedly in time until finally the most northerly regions are involved quite strongly in the warming phase of a wave. When this happens, the entire arctic and sub-arctic areas warm rapidly, over the layer from, say, 10 mb or less to 200 mb, to temperatures comparable to those of mid-summer. After this final warming, some cooling may take place but no appreciable temperature gradients or temperature waves are observed. The baroclinic characteristics of the winter stratosphere give rise to the arctic stratospheric jet stream, in which wind speeds exceeding 200 kt have not infrequently been observed at very high levels. This jet stream has disappeared at the end of the rapid warming event, following which the arctic stratosphere is rather inactive, both thermally and dynamically.

Although this final warming is often referred to as the abrupt winter-summer transition of the arctic stratosphere, it does not necessarily usher in the weak easterly monsoon of mid-summer. Actually, there appear to be two main types of winters – the early final-warming winter, with an especially intense event at very high levels, and the baroclinic-wave winter, with a relatively late final warming. In the former case, there is only one really pronounced baroclinic wave, which engulfs the entire stratosphere in depth and the entire polar regions in extent. This occurs in January or early February, and is followed by a return to relatively weak westerly flow. The return of the sun reverses the radiative cooling before pronounced baroclinicity can be created, and a gradual transtion to summer conditions takes place. The second type of winter is characterized by frequent moderate-amplitude baroclinic waves. Their intensities may decrease, increase or remain unchanged, but one particular wave, in March or early April, terminates the series by acting as a final-warming wave. In this case, the transition to summer conditions is generally permanent. The great variability from year to year is one of the more important aspects of the behaviour of the arctic winter stratosphere in the present context.

In both types of winters, the final warming may move from west to east (the usual event) or from east to west (roughly one year in seven). Studies of these waves (and of earlier baroclinic waves in the various winters) reveal the primary role played by active regions in the stratospheric warm belt (Kochanski 1955), especially the Alaska-Aleutian area. These regions appear to intensify baroclinic waves by supplying a source of warm stratospheric air. It is generally admitted that these high temperatures are ascribable to large-scale subsidence.

3. General relations between total ozone and stratospheric temperatures

It has long been known that there is, in general, a good positive correlation between total ozone and stratospheric temperatures. This was first demonstrated by Meetham (1937) who found a maximum correlation with potential temperature at 18 km (the highest level he used), over the period March to November. This he ascribed to similar effects of horizontal advection and vertical motion on potential temperature and on ozone, although considerably more ozone exists above 18 km than below. Penndorf (1950) showed the importance of 97 mb trajectory for total ozone changes. On the other hand, Normand (1951) found that, in mid-winter, the correlation between total ozone and stratospheric temperature may increase or decrease with height for periods of several weeks' duration, strongly suggestive of separate lower and middle stratosphere regimes. Ozone correlations with troposphere and lower stratosphere parameters have been explained by Reed (1950) on the basis of the fields of advection and vertical motion customarily ascribed to the baroclinic waves of the troposphere and lower stratosphere. As shown by Normand (1953) and Johansen (1955), these correlations remain high for long waves as well.

However, these correlations do not appear able to shed any light on the rapid ozone increase of late winter, which is, characteristically, not a short-period phenomenon. We are, therefore, led to examine the long-period relations between total ozone and middle stratosphere temperatures, since during the period of interest the middle stratosphere, especially at high latitudes, is dominated by long-period baroclinic waves. The first problem is how to represent the long-period variations. The solution adopted was to evaluate 10-day running means of total ozone (in atm-cm, converted where necessary to the Vigroux scale) and of 100 mb temperature. Missing values were estimated wherever possible, by interpolation and/or extrapolation. The use of 10-day running means as a filter on a time sequence subdues almost completely variations with a period of 10 days or less, and reduces the amplitude of variations with periods in excess of 10 days. For a 15-day period, the amplitude reduction is 59 per cent, for a 20-day period 36 per cent, for a 25-day period 24 per cent and for a 30-day period 17 per cent. This procedure should effectively filter out the major effects of the lower stratosphere on total ozone variations, leaving only the effects of processes in the middle stratosphere or of long-period processes in the lower stratosphere which are probably highly correlated with middle stratosphere behaviour. By itself, temperature at the 100 mb level is truly representative of neither the lower nor middle stratosphere since in winter this level lies near the boundary between the two regimes. However, the time filter subdues the short-period variations which are lower stratospheric in origin and, therefore, accentuates the middle-stratospheric components. As pointed out by Godson and Lee (1958), for historical surveys of data over the past ten years or more, this level is actually the highest that may realistically be employed. Although the time filter employed on the total ozone and 100 mb temperature does tend to isolate the features of chief interest (the long-period variations), one must realize that basically we are comparing a quantity integrated over the entire atmosphere (total ozone) to one pertaining to a specific discrete level. Nevertheless, as long as we bear this limitation in mind, we may imagine that primarily we are comparing the bulk of ozone (that in the middle stratosphere) to temperature in the middle stratosphere.

The comparisons to which we have just referred will be presented graphically in later sections, over the period September to May for a number of representative recent winters. However, in order to have a complete framework within which the results may most profitably be discussed, it will be helpful to produce, in advance, a rather complete hypothesis concerning the nature of the relations between middle-stratosphere ozone and middle-stratosphere temperature in general, and between 10-day running means of total ozone and 100 mb temperature in particular. Once again, we shall restrict our attention to the period September to May; during most of this time the middle stratosphere is baroclinic and dynamically active. Confirmatory evidence for many of the particular aspects of the hypothesis will, of course, be presented later, but fundamentally we need

only know that there is a close relation between the two parameters under discussion. Earlier correlation studies would suggest this premise and we shall find that, in general, the agreement over the entire period between total ozone and 100 mb temperature is very good, when short-period variations ascribable to lower stratospheric activity have been filtered out of the time series. One may conclude that when and where the 100 mb temperature is high (low), the total ozone is also high (low). Moreover, in arctic and sub-arctic areas at least, total ozone is low and stays low from September to December, increases rapidly during the period of the middle-stratosphere final warming, then levels off and decreases slowly in late spring and in summer.

The physical relations between total ozone and 100 mb temperature are, of course, rather complex. We shall assume that in general the filtered series at 100 mb is representative of the middle stratosphere thermal regime, where troughs are cold and ridges warm and vertical motions are opposed to those in the lower stratosphere (Kochanski 1954). On the average, a rising temperature means warm-air advection plus ascent whereas a falling temperature means cold-air advection plus subsidence. For purposes of discussion we shall assume that over the period of good correlation (we definitely exclude summer since for that period there is no disturbed middle stratosphere) there is only a single high-level maximum ozone mixing ratio and the vertical distribution is sufficiently specific to total ozone that high total ozone means high ozone mixing ratio in the middle stratosphere. Thus warm-air advection in the middle stratosphere should be associated, on the average, with a positive ozone advection while cold-air advection should accompany a negative ozone advection. Moreover, ascent causes an ozone as well as a temperature decrease in the middle stratosphere, on the average, and subsidence should produce increases in both. The final link in the relation is provided by the assumption that, in general, ozone and temperature gradients in the horizontal and vertical are such that when vertical motion is opposed to advection and of less significance for temperature changes, the same holds true for ozone changes.

It will be quite clear that the above assumptions are quite arbitrary, but, nevertheless, they do provide a framework inside which we may discuss departures from good correlation. It will, of course, be evident that there is no satisfactory alternative to adequate data on the vertical distribution of ozone, available synoptically from a suitable network of stations. All that can be done by the present class of studies is to pose questions and draw attention to major features or events. If we had a good ozonesonde network, bolstered by an expanded programme of high-level rawinsonde flights (e.g., to 10 mb) and reinforced by a maximum *umkehr* programme at all spectrophotometer stations, it would then be possible to explain all variations on a day-to-day basis, and thus to arrive at explanations for the salient features on a longer period, including seasonal effects.

For the present, it appears to be helpful to consider in some detail cases and causes of major disagreement between running means of ozone and of 100 mb temperature. These seem to shed more light on the processes at work than the case of agreement, for which we have already formulated a working hypothesis. There are many possible causes of disagreement. In the first place, the very-long-period (or seasonal) changes are not in phase. Temperatures in the middle stratosphere have a minimum in January, on the average, whereas the ozone minimum is more nearly a flat valley from September to December, with a definite ozone increase prior to that associated with the final warming of the middle stratosphere. Moreover, there is no ozone maximum associated with the flat plateau of summer stratospheric temperatures, although there definitely is an ozone maximum at the time of the winter-summer transition of the arctic and sub-arctic stratosphere. These differences arise in large measure from the different responses of temperature and ozone to radiation balance, and in particular to the spectral details of this balance. The responses of temperature and ozone to the transfer characteristics of the general circulation, on the other hand, are probably rather similar. Thus, for example, the strong cooling in the autumn of the middle stratosphere, being predominantly an infra-red feature, produces no analogous decrease in total ozone. At the time of minimum tempera-

tures, in January, the general circulation must supply heat to the sub-arctic and arctic areas to balance the radiative cooling. This is associated with an ozone transfer resulting in a general increase in total ozone. The rapid warming of the stratosphere at northern latitudes, whether advective in origin or due primarily to subsidence, is associated with a comparable rapid increase in ozone.

The summer maximum of solar radiation can increase stratosphere temperatures above those typical of the mild cooling following the final major baroclinic wave since the infra-red cooling increases only slightly as the temperature increases. On the other hand, it must be presumed that the vigorous advective and/or dynamic ozone increase to high values in the middle stratosphere has overshot the photochemical equilibrium values, so that net destruction occurs in summer at high levels in addition to the transfer to the south as a result of inter-latitudinal exchange. In temperate latitudes there is no anomalous winter-summer transition in the stratosphere, although processes in the sub-arctic middle stratosphere often have briefer and less-intense analogues further south (Scherhag 1952), usually confined to very high levels (e.g., 10 to 25 mb), and to periods of strong baroclincity, i.e., until the final stratospheric warming at high latitudes has spread over all longitudes.

It follows from the above that the major source of the spring increase in total ozone must lie in the migratory warm belts of the sub-arctic middle stratosphere, the ozone descending by subsidence and then being transferred northwards by the baroclinic waves associated with the arctic stratospheric jet stream. It appears that there are favoured areas for this injection of ozone and enthalpy into arctic areas – principally the Aleutian region and in some years the North Atlantic area. Both of these areas are downstream from regions of intense tropospheric jet-stream cores – the Japan and Newfoundland sectors, respectively – and the initial subsidence may be linked to the rapid downstream decelerations in the diffluent streams to the east of jet maxima (Wege *et al.*, 1958; Teweles 1958).

The net effect of these seasonal phase differences between middle-stratosphere temperature and total ozone is that maxima and minima may be subdued or distorted during periods of temperature (but not ozone) decreases or of ozone (but not temperature) increase. Similar distortions can be produced by variable effects of short-period oscillations, especially those with periods of about one to two weeks.

A second reason for disagreement is the possible unrepresentativeness of the 100 mb level for middle-stratosphere phenomena. It is admitted that the 50 mb level would probably be preferable for these studies, but for earlier years these latter data are largely nonexistent. Frequently the major changes in the mid-stratosphere occur in the 10 to 25 mb layer, even in arctic and sub-arctic areas, and we shall be able to see examples where these latter layers correlate better with total ozone than does the 100 mb level. At times, the 100 mb level may be dominated by lower stratosphere disturbances (early autumn and late spring) and although the filtering process removes short-period changes of such origin the residual field may be only a weak reflection of the true behaviour of either the lower or the middle stratosphere.

Finally, many of the necessary details in the primary correlation hypothesis may at times fail to be valid – the relative effects of horizontal and vertical advection on temperature and on ozone, the single high-level maximum of ozone mixing ratio, or the closely in-phase behaviour of isotherms and ozone mixing-ratio isopleths. Without ozonesonde data, it would be very difficult to isolate a single cause of disagreement in this category.

As examples of the processes just discussed, we shall now analyze actual data for four representative winters – two of the early final-warming variety (1957-58 and 1956-57) and two of the baroclinic-wave type, with a late final-warming (1955-56 and 1953-54). Data for Canadian and northern European stations (of ozone and 100 mb temperature) have now been analysed for nine winters; the remaining five winters, not discussed here, also substantiate the mechanisms suggested.

4. 1957-58 WINTER

The behaviour of the stratosphere in arctic and sub-arctic areas during the 1957-58 winter has been described by several authors, notably Teweles and Finger (1958); Zubyan (1959); Krishnamurti (1959); Pogosyan (1959). Briefly, 1957-58 was a winter with an early final warming, very intense at high levels, yet without pronounced baroclinic waves prior to this time. At arctic latitudes, the final warming was initiated over eastern Siberia and western Alaska, being more intense over Alaska. The warming advanced rapidly eastwards and increase in intensity, with a maximum intensity over the arctic Archipelago in late January, subsequently weakening as it passed over Greenland. Deceleration occurred over the North Atlantic, and the original single-warming event became a slower stepwise process by the time it reached Scandinavia. At levels above 100 mb, the phenomena were even more complex, as we shall see later.

In Fig. 2 are shown the 10-day running means of 100 mb temperature and of total ozone for four Canadian stations – Alert (82·5°N, 62·3°W), Edmonton (53·6°N, 113·5°W), Moosonee (51·3°N, 80·6°W) and Resolute (74·7°N, 95·0°W). For Resolute and Alert, only fragmentary ozone data were available through the winter months, but it seemed worth while to utilize all available information in view of the general interest in such northern latitudes. Accordingly, short periods of data on the moon as source of reflected ultra-violet radiation were averaged to give a single data point, and longer sequences of data (sun or moon) were averaged to give 5-day running means. These will be compared to 10-day running means of 100 mb temperature.

During the autumn months of 1957, ozone at Alert averaged about 0·02 cm less than at Resolute, and the two seemed to rise and fall together with relatively great variations, considering the almost monotonic temperature decrease with time in the middle stratosphere. During the rapid-warming and ozone-increase transition from winter to summer, Resolute appeared to have commenced the ozone increase before Alert, Alert catching up in early February and rising to a higher value in early March. By May, total ozone amounts at the two stations were nearly identical.

The mid-October minimum and the mid-November secondary minimum and maximum (respectively) for the smoothed 100 mb temperature at Alert seem to be reflected in the few ozone values obtained. The latter changes (in ozone) appear to be relatively more intense, since there is no strong background decrease in ozone comparable to the strong radiative cooling which tends to swamp the wave-evolution of the thermal field of the middle stratosphere at high latitudes. The very rapid increase in ozone coincides with the explosive warming of late January. During May, ozone changes seem more vigorous than could be explained by the middle stratosphere alone, but this period is part of the summer monsoon when only the lower stratosphere is dynamically active.

At Resolute, total ozone and 100 mb temperature are very well correlated from early September to mid-October. In early November and early December the data are reasonably consistent. The early January ozone increase seems anomalous, but as we shall see this is the general period of maximum disagreement. The explanation appears to be that, while advection and vertical motion balance with respect to temperature changes, they will not do so for ozone since at that time the effects of vertical motion (on ozone) are relatively greater than those of ozone advection. This is at least partially borne out by the studies of Martin and Brewer (1956) who showed that, over northern Europe in December 1952, the correlation between changes in total ozone and in 100 mb temperature was decidedly smaller than that between changes in total ozone and the average vertical motions at 100 mb. This is borne out by the results of a later study (Martin and Brewer 1959) which gave comparable values for these two correlations during September 1953. Data for early February and early March at Resolute are very concordant, as would be expected. Spring ozone data would be anticipated to reflect primarily lower-stratosphere processes, only weakly in evidence at the 100 mb level.

Moosonee and Edmonton are situated approximately in the winter warm belt, as far as middle-stratosphere temperatures are concerned, with the cold arctic stratosphere to

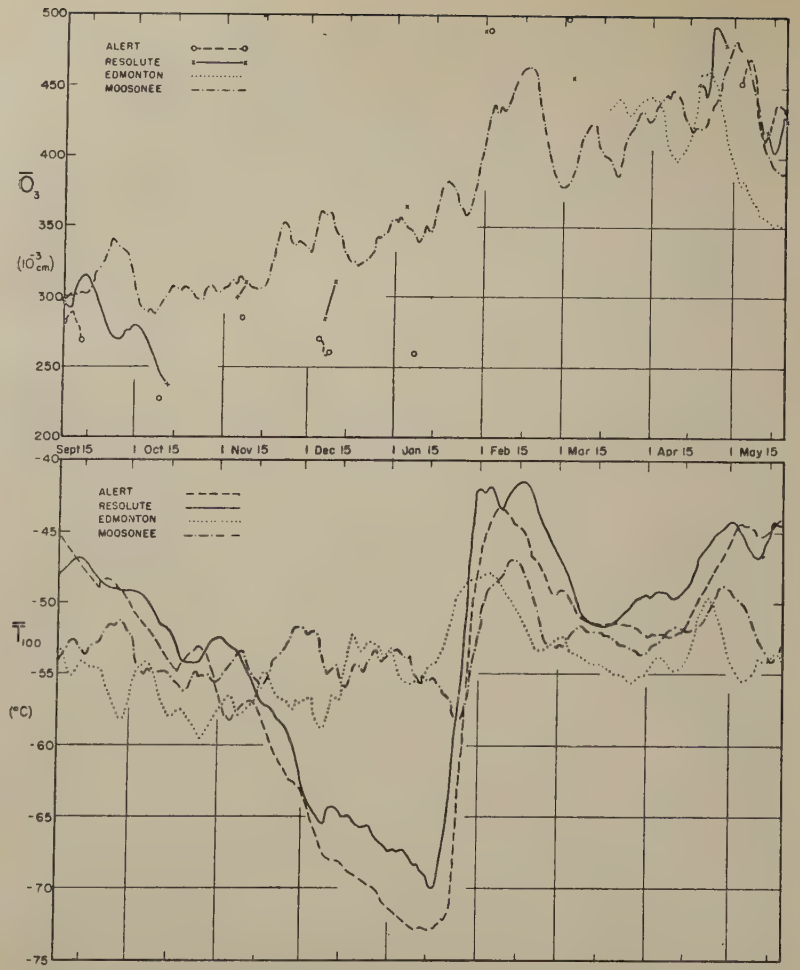

Figure 2. 10-day running means of 100 mb temperature and of total ozone for 1957-58 winter at Canadian ozone stations.

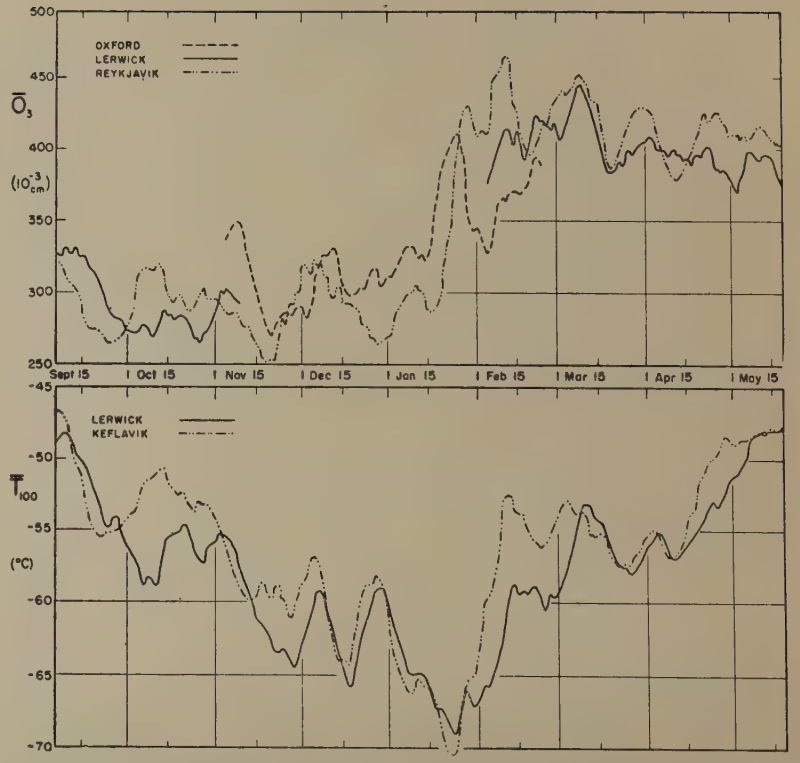

Figure 3. 10-day running means of 100 mb temperature and of total ozone for 1957-58 winter at European ozone stations near 60°N.

the north and the cold tropical stratosphere to the south. This is also the belt of maximum ozone, except for a period in late winter and in spring when the maximum lies much farther north. The Edmonton ozone data are for the period after the final arctic-stratospheric warming (quite evident at 100 mb at Edmonton in late January), and presumably after the main ozone maximum. The agreement of ozone with 100 mb temperature is relatively good, especially in late April, although the agreement is somewhat better with 200 mb temperature (a general feature of the summer monsoon period).

At Moosonee, the most prominent features on both curves are the late January and early February maximum. After this time, the lower stratosphere takes over as the dynamically-active region, and only its features which strongly affect the 100 mb level produce good correlations between ozone and 100 mb temperature, e.g., the late April maximum and the mid-May minimum. Between mid-September and mid-January, the agreement is fair, although the period from mid-November to mid-January is rather poor in this regard (a general result in all areas). It may be recalled that even further north there were no strongly baroclinic waves during this period of this winter, so that a strong linkage between total ozone and the middle stratosphere would not be expected.

We may now examine comparable data for northern European stations. West-east variations in total ozone and in 100 mb temperature, both evaluated as 10-day running means over the period from early September to mid-May, are illustrated in Fig. 3 by curves for Keflavik (Iceland) and Lerwick (Scotland), Oxford ozone data being included for the period in mid-winter when ozone values at Lerwick were not published. At Keflavik, the agreement between 100 mb temperature and total ozone is excellent from September to mid-December and from February to early May. The mid-December to mid-January anomalous period is not a result of the 100 mb level being unrepresentative of the middle stratosphere, since higher levels behave similarly. It seems most likely, since the ozone minimum at the end of December coincides with a temperature maximum, that here ascent under-compensates temperature advection but over-compensates ozone advection. This is most likely to occur if a relatively strong positive lapse rate exists in the stratosphere and least likely if an inversion exists. For this particular baroclinic wave, the high-level temperatures at Keflavik remained lower than those at 100 mb. There may have been significant differences in the isobaric advection of temperature and of ozone – such as minimum gradients of ozone at this time (the end of the broad valley and the start of the slow rise).

The late January ozone increase appears at first sight to be anomalous but this event can be readily explained. This is a case when the major activity in the middle stratosphere took place at very high levels (10 to 25 mb) in a very abrupt manner, compared to the 100 mb level where changes were less rapid and represented more nearly one single final-warming event. Typical 25 mb temperatures during this period were : 25 January 00 GMT, — 72°C; 29 January 00 GMT, — 33°; 29 January 12 GMT, — 19°; 31 January 00 GMT, — 76°; during the early February warming episode the 20 mb level reached — 26° by 10 February and had cooled to — 49°C by 13 February. During the next two months the temperatures in the layer 10 to 20 mb remained very near — 60°C, which demonstrates that after the completion of the final warming the major activity in the stratosphere shifts from the middle stratosphere to the lower stratosphere, as a prelude to summer conditions during which the layer 10 to 100 mb is extremely inactive, dynamically or thermodynamically.

The temperature changes over Keflavik during this period are relatively similar to those that occurred over the North American continent at about the same time, and are indicative of very large vertical motions in the layer from 20 to 30 km. In the case of Keflavik, an isolated isotherm maximum from the warm belt moved north-westward from southern France and northern Italy at 30 mb on 24 January toward Keflavik, intensifying rapidly. This temperature maximum later moved south-westward over the North American continent, reaching San Diego (32·5°N, 117°W) on 4 February, with a 10 mb temperature at 00 GMT of — 40°C. Three and a half days later the pronounced minimum temperature

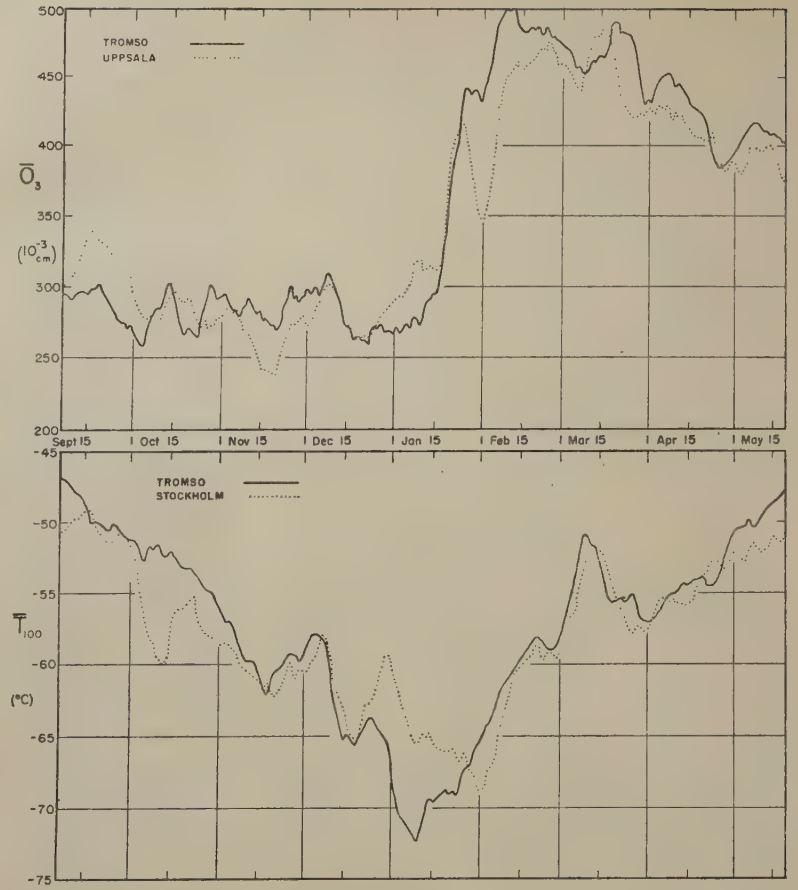

Figure 4. 10-day running means of 100 mb temperature and of total ozone for 1957-58 winter at Scandinavian ozone stations.

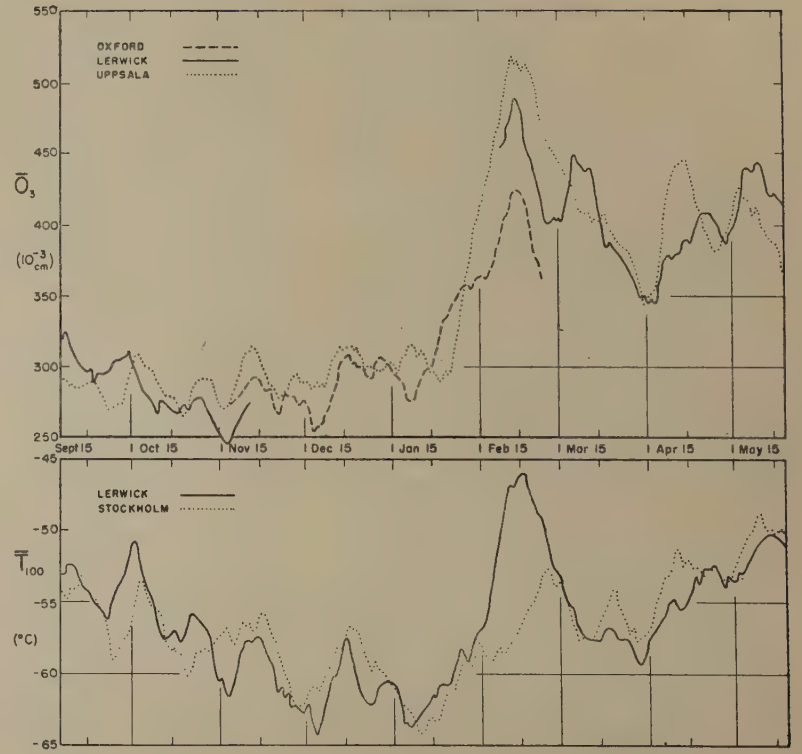

Figure 5. 10-day running means of 100 mb temperature and of total ozone for 1956-57 winter at European ozone stations.

wave was passing over Denver (40°N, 105°W) with a 15 mb temperature at 12 GMT of — 67°C.

Stations in north-west Europe also displayed these relatively large fluctuations in high-level temperatures, but not to the extreme extent seen at Keflavik. Some examples of the high temperatures at high levels in late January, while the 100 mb level was still relatively cold, were : Lerwick, — 47°C at 47 mb on 30 January, and Helsinki, — 58°C at 41 mb on 25 January.

The variations in total ozone at Lerwick (and at Oxford) are also well-phased with the temperature at the 100 mb level, with the exception of periods in late December and late January, which have already been explained. Over the United Kingdom the final stratospheric warming was not completed until early March, at which time the major ozone maximum occurred. It appears that in this region the early February warming was not especially intense at very high levels.

The Scandinavian stations of Tromso and Stockholm, whose data are shown on Fig. 4, display very similar features – relatively good agreement of total ozone and 100 mb temperature from September to mid-December, then poor agreement until early February for the reasons previously cited. The more rapid and intense ozone increase in February relative to that in 100 mb temperature can be explained by the fact that the temperature changes in the middle stratosphere above 100 mb occurred earlier and were more pronounced. Thus, for example, Ostersund had a 32 mb temperature on 10 February of — 42°C, when the 100 mb temperature was — 62°C. Similarly, at Tromso a temperature of — 53°C was observed at 35 mb on 10 February, and at 60 mb not until 12 February. During the spring months the agreement is not particularly good since the final warming has been completed and the middle stratosphere is no longer a dynamically active layer. Instead, the 100 mb level is merely a diluted indicator of processes in the lower stratosphere, which probably now contains much more ozone than it had in January.

5. 1956-57 WINTER

The behaviour of the stratosphere in arctic and sub-arctic areas during the 1956-57 winter has been very thoroughly documented, for example by Craig and Hering (1959), Teweles (1958) and Godson and Lee (1958). Briefly, 1956-57 was also a winter with an early final warming, very intense at high levels (above 100 mb) but preceded by baroclinic waves of greater amplitude than in the following winter. At high latitudes, the final warming at 100 mb commenced in late January in eastern Siberia (near 150°E) and moved rapidly westwards, being reinforced by a secondary source over the North Atlantic (the primary source at high levels and for most sub-arctic areas).

Fig. 5 presents ozone and 100 mb temperature data for European stations. During this winter, total ozone at Stockholm and at Lerwick (plus Oxford) is in good agreement with the 100 mb temperature with the exception of early March at Lerwick and both February and March at Stockholm. The general features of this winter's ozone variations are well described in terms of the final stratospheric warming in February. Prior to this period, ozone showed a broad minimum in late fall, a slow increase into January and a rapid increase at the time of the final warming to the major maximum, and was followed by a general decrease.

The Lerwick ozone maximum in early March appeared to have been associated with a warming of the lower stratosphere, which was damped out by 100 mb, in accordance with the view that the lower stratosphere takes over as the seat of major ozone changes after the winter's final warming has been completed.

The sharp ozone maximum at Stockholm in mid-February is explainable on the basis of an earlier and more intense warming above the 100 mb level. Although Stockholm upper-air data were very poor during this month and March, Ostersund and Tromso exhibited such a behaviour, and in addition showed a much weaker maximum in mid-March. Since final warmings generally increase in intensity with height, it is particularly important to have available rawinsonde ascents to high levels, since extrapolation in the vertical will underestimate the rapidity of such final warmings.

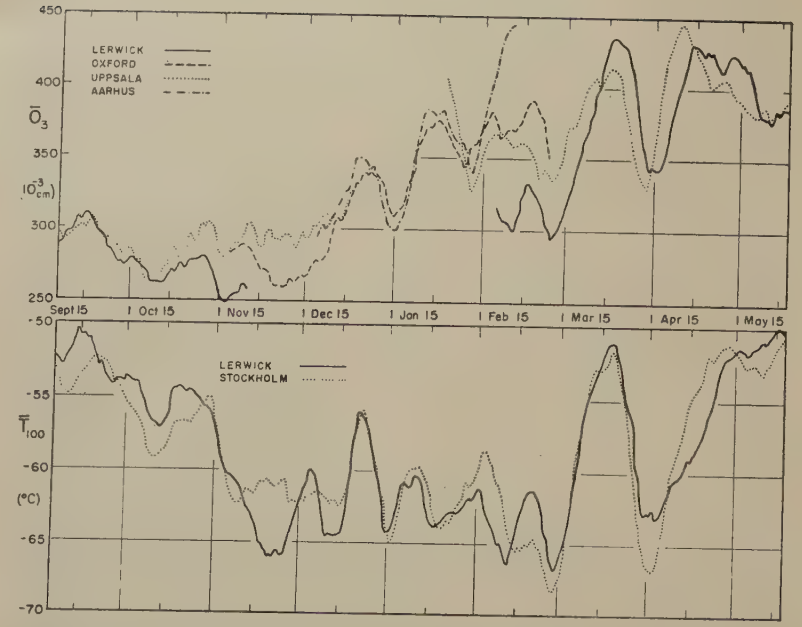

Figure 6. 10-day running means of 100 mb temperature and of total ozone for 1955-56 winter at European ozone stations near 60°N.

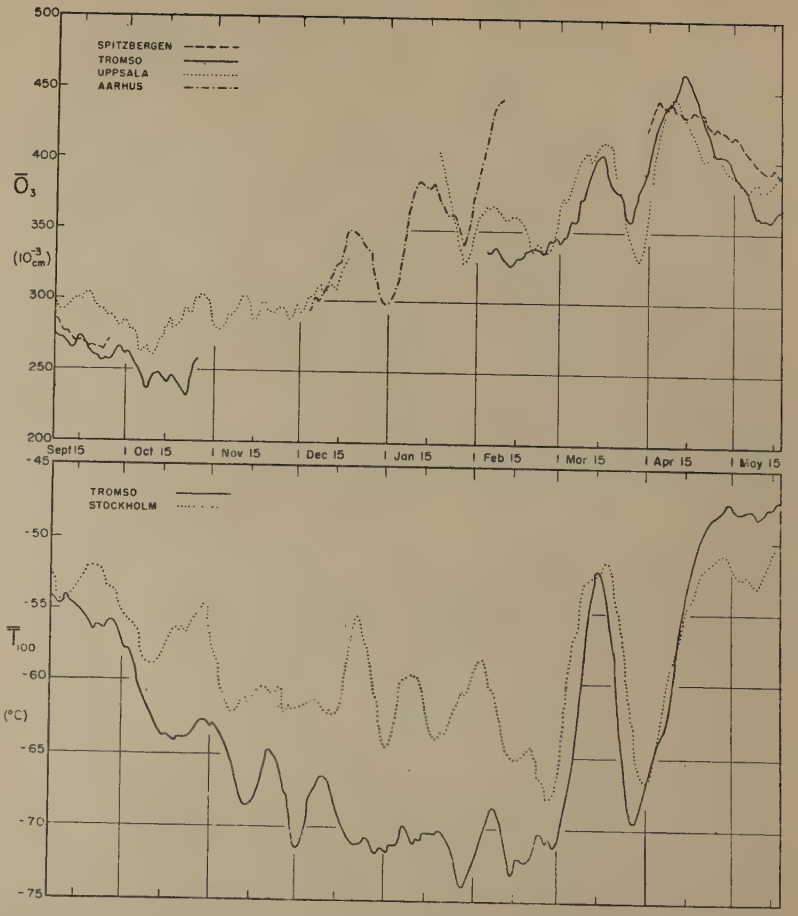

Figure 7. 10-day running means of 100 mb temperature and of total ozone for 1955-56 winter at Scandinavian ozone stations.

6. 1955-56 WINTER

The behaviour of the stratosphere in arctic and sub-arctic areas during the 1955-56 winter has been dealt with by Gaigerov (1957), Lee and Godson (1957) and Godson and Lee (1958). This winter is best characterized as a baroclinic-wave winter, with two major events which moved from west to east – a strong baroclinic wave which reached Europe in mid-March and the final-warming wave which arrived in early April, both originating in or to the west of Alaska. Ozone and 100 mb temperature data for European stations are shown in Figs. 6 and 7.

In Fig. 6, we have data for stations near 60°N – Lerwick (using Oxford ozone for the mid-winter period) and Uppsala (using Aarhus, Denmark, to complete the winter). For the British stations, the chief periods of disagreement, comparing ozone and 100 mb temperature, are : early November, early December, mid-January, mid-April and May. In early November the strong radiative cooling tends to suppress the temperature maximum, but it was evident on daily values and was slightly more intense at higher levels. The temperature maximum in early December was much weaker at Ship M, at Ostersund and at Stockholm than at Lerwick and may have been in part the result of doubtful extrapolations. The out-of-phase relationship in mid-January has no simple explanation, but is related to the general disagreement that frequently prevails in December and January, especially when temperature waves are weak in amplitude and baroclinicity. The ozone maximum in mid-April was probably associated with a more intense and rapid warming at higher levels than at 100 mb. Although no data were available at Lerwick above 50 mb, at Keflavik the major warming occurred in the 15 to 20 mb layer in mid-April, while the 100 mb level was cooling slightly. During May, after the final warming, the 200 mb level appears to be better correlated with total ozone than the 100 mb level, as is normal at this time.

For the southern Scandinavian stations the chief disagreements are in the timing of maxima and minima in late January and in April. The former is the normal situation and the latter is associated with an earlier and more intense warming at 50 mb, probably even more so at higher levels. Similar effects are seen further north, for example at Spitzbergen and at Tromso (Fig. 7). The total ozone at Tromso shows relatively little reaction to the last complete temperature wave (prior to the final warming), an event duplicated in the 1953-54 winter, which also had major baroclinic waves throughout the winter. At Keflavik (where high level data were particularly good) the major March maximum was earliest at 200 mb, and the amplitude decreases with height – precisely the opposite to the behaviour during the final warming in April. Tromso data during April agree with those at Ship M and Ostersund in showing an earlier and more intense warming at high levels. Typical data at Tromso during this period were : on 5 April, — 65°C at 100 and at 70 mb; on 11 April, — 61°C at 100 mb and — 54°C at 55 mb; and on 13 April, — 57°C at 100 mb and — 40°C at 40 mb. At high levels the temperature levelled off near — 40°C and was actually falling during late April while temperatures at 100 mb were still rising.

7. 1953-54 WINTER

The evolution of the stratosphere during the winter of 1953-54 has been discussed by Warnecke (1956) and Austin and Krawitz (1956). This winter can best be described as one of frequent temperature waves of moderate amplitude and baroclinicity. These waves moved from west to east and culminated, without any appreciable trend in intensity, in a final-warming wave which crossed the Canadian Arctic in early March and reached Europe in mid-March.

10-day running means of total ozone and 100 mb temperature for Edmonton, Canada, are shown in Fig. 8, supplemented by temperature data for Norman Wells, 12 latitude degrees NNW of Edmonton. The Edmonton 100 mb temperature (and ozone also) are in relatively good agreement with the thermal field of the middle stratosphere further

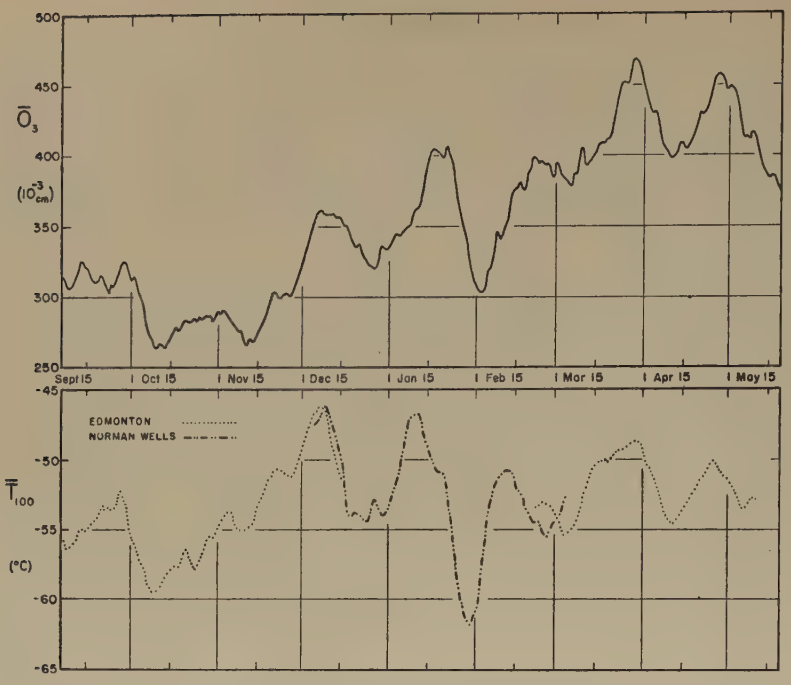

Figure 8. 10-day running means of 100 mb temperature and of total ozone for 1953-54 winter at Edmonton, Canada.

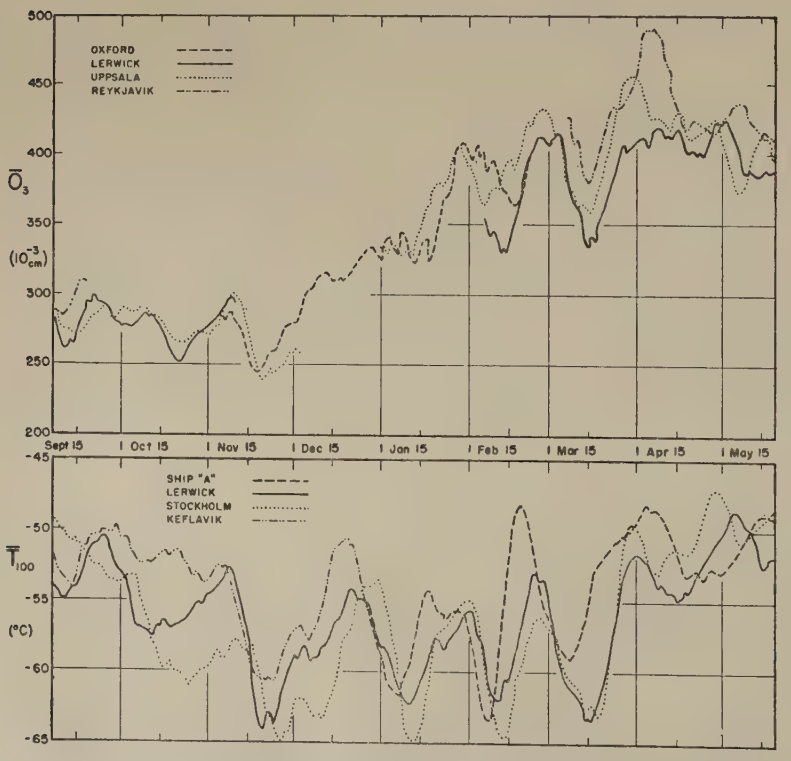

Figure 9. 10-day running means of 100 mb temperature and of total ozone for 1953-54 winter at European ozone stations near 60°N.

north in the true sub-arctic – where one observed the final warming in the first half of March, preceded by the major minimum at the end of February, with a major maximum in early January, followed by a weaker maximum in mid-February, after a minimum in early February. There is no real ozone minimum at the time of the temperature minimum in early March, but this is fairly characteristic of a baroclinic-wave winter with stepwise increases in ozone before the final warming. It appears that the agreement between these two parameters is somewhat better for stations in the latitude range of the stratospheric warm belt than for stations farther north, but this may be true only in the vicinity of active regions of the warm belt, where the primary production of warm ozone-rich stratospheric air masses takes place.

The corresponding curves for European stations are shown in Figs. 9 and 10. In the vicinity of 60°N (Fig. 9), the temperature data demonstrate the eastward motion of the baroclinic waves, up to and including the final-warming wave of late March. With the institution of the summer stratosphere monsoon at this time, waves later moved westward, with a very much reduced amplitude. Ozone variations follow quite a reasonable pattern – a broad minimum during September to November, a gradual increase through January and February associated with the vigorous baroclinic disturbances of the Arctic stratospheric jet stream, and a major maximum at the time of the final warming.

The agreement between 10-day running means of total ozone and of 100 mb temperature at Keflavik, at Lerwick (plus Oxford) and at Stockholm is exceedingly good, with the exception of early January. This is the period of greatest positive lapse rate in the middle stratosphere and therefore of the minimum compensation by subsidence to cold-air

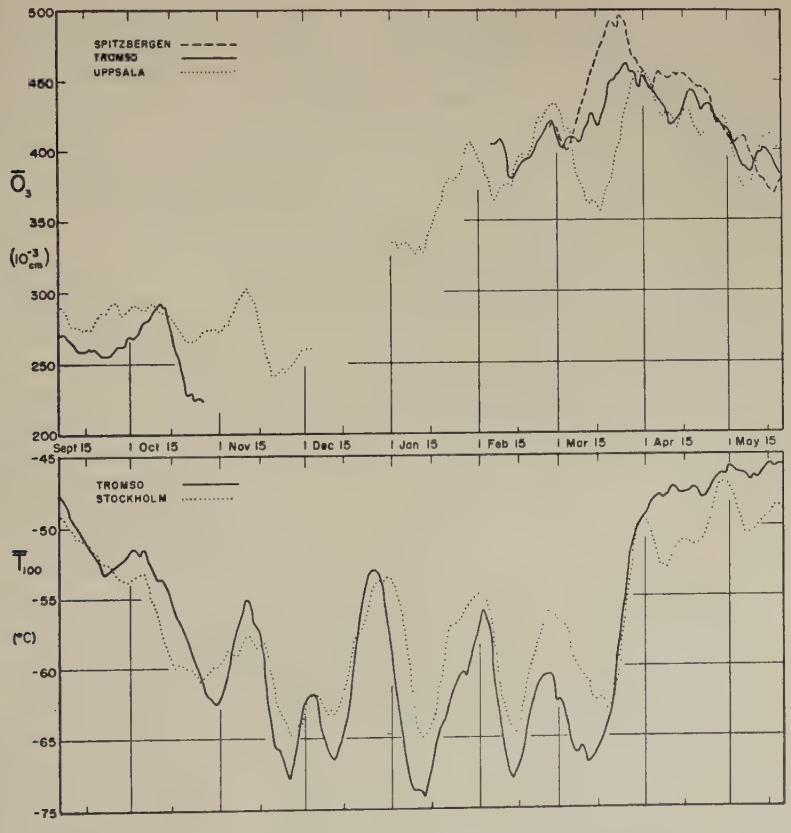

Figure 10. 10-day running means of 100 mb temperature and of total ozone for 1953-54 winter at Scandinavian ozone stations.

advection. Thus, if ozone gradients are simultaneously a minimum, middle-stratosphere cooling will not be associated with a marked ozone decrease as long as vertical velocities are moderate and negative. Minor discrepancies after the final warming can once more be ascribed to the shift of dynamical and ozone activity from the middle to the lower stratosphere.

Similar features are displayed at the three Scandinavian stations, Spitzbergen, Tromso and Stockholm (Fig. 10). The earlier and stronger ozone maximum at Spitzbergen is in excellent agreement with the corresponding 100 mb temperature maxima at the North Greenland stations Nord and Scoresbyssund (not shown in Fig. 10). The relatively weak response of ozone at Tromso to the strong temperature waves appears to be associated with a decrease in amplitude of the latter with height. Although high-level data were not available for this winter at European stations, an examination of the soundings at Ship *A* (60°N, 30°W) shows a definite decrease in activity to 50 mb. This is probably fairly typical of winters with frequent, large-amplitude temperature waves and a relatively late final warming. On the other hand, winters with weaker temperature waves at 100 mb generally show more intense activity above 100 mb and an earlier final warming.

8. Conclusions

The basic explanation of the spring increase in ozone is thus linked to that of the winter-summer abrupt transition in the arctic stratosphere. Unfortunately, the precise mechanisms involved in the latter phenomenon are still rather obscure. It is certain that the lower and middle stratosphere at high latitudes cools radiatively most, if not all, of the year. The annual temperature cycle thus has a strong general circulation component involving a convergent meridional enthalpy flux. This does not necessitate a meridional steady circulation since in winter, in particular, large-scale baroclinic waves which have efficient transfer mechanisms are a concomitant of the existence of the arctic stratospheric jet stream. Heat energy and total ozone are thus provided in spurts, at any one longitude, and the final baroclinic wave, generally rather complex and very intense, is sufficient to produce summer stratospheric temperatures at high latitudes and the annual ozone maximum as well. This explosive behaviour destroys the jet stream, and the circulation soon degenerates into the easterly monsoon of summer.

At one time it was believed that stratospheric circulations, above the levels of tropospheric compensation, were in general rather sluggish and that, as a consequence, horizontal and vertical transfer through the stratosphere would necessitate mass transport, or steady mean circulations in vertical planes. We now realize that, especially in winter, the middle stratosphere is dynamically active, and that transfer can very simply take place by means of horizontal and vertical fluxes, the individual eddies being primarily on the synoptic scale. Total ozone is increased if there is a net downward flux from levels in photochemical equilibrium to protected levels where ozone is a conservative property. This process takes place in active regions of the stratospheric warm belt (latitudes near 50°) throughout the winter. Intermittently, the warm subsided ozone-rich stratospheric air mass is injected into the baroclinic waves of the arctic stratospheric jet stream, the maximum injection taking place with the final-warming wave. By this time, there will be large ozone concentrations in both the lower and the middle stratosphere. Ozone maxima at latitudes relatively unaffected by the final-warming wave will be delayed, since they are produced primarily by advection from the north during low-index circulation regimes after the maxima at high latitudes have been established. Wexler (1958) drew attention to the evidence for such a mechanism during the 1951-52 final warming over Edmonton, and Wexler and Moreland (1958) correctly assessed the synoptic situation accompanying early ozone maxima at Tromso in terms of the processes outlined above. The present hypothesis is thus not a new one, but the quantitative comparisons of this paper were required in order to weld into a coherent picture the suggested ozone mechanism and our new concepts of the behaviour of the arctic winter stratosphere.

Acknowledgment

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Some computations of the variation of vertical velocity with pressure on a synoptic scale

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(Manuscript received 5 November 1959)

SUMMARY

The vertical velocity in pressure co-ordinates is computed from synoptic data on two occasions for each 100 mb interval between 1,000 mb and 200 mb at the points, approximately 160 mi apart, of a mesh covering much of the Atlantic Ocean and Europe. The assumptions made are that the mean lapse rate is everywhere the same, that there is little movement of air through the upper and lower pressure surfaces and that the horizontal wind is approximately geostrophic. The results show that the vertical profile is approximately parabolic with respect to pressure in regions of great development, but that elsewhere the profiles are not of a simple shape.

1. INTRODUCTION

It is well known that vertical velocities cannot be measured accurately in the atmosphere except with special equipment at a few isolated points and that so far it has not been possible to make routine measurements to aid in the problem of synoptic forecasting. Values of the vertical velocity have been inferred from synoptic observations by several writers, using a variety of methods (Panofsky 1951, 1957; Bushby 1952; Masuda 1950; Mashkovich 1956; Fleagle 1958). On the whole it may be said that their results, obtained by different methods applied to the same synoptic type, give the same qualitative distribution of vertical velocity.

The two-parameter baroclinic models which have been used for numerical prediction have usually assumed a parabolic or sinusoidal form for the vertical variation of the rate-of-change of pressure on a moving air parcel, for the variation can depend only upon one parameter and either form represents crudely a maximum in the middle troposphere falling to near zero in the upper and lower tropospheric levels. The object of this investigation was to compute the distribution in the vertical of rate of change of pressure, dp/dt, using data from more than two levels in the atmosphere.

2. THE BASIS OF THE COMPUTATIONS

Most of the dynamical models which form a basis for numerical prediction of the pressure distribution proceed from two basic equations; that which describes the changes in vorticity with time, and that which describes the changes of temperature with time. The two equations are ultimately linked through the vertical velocity which is necessary to balance horizontal divergence and which also leads to warming or cooling. In pressure co-ordinates the equations may be written in the usual notation :

$$\frac{\partial \zeta}{\partial t} + \mathbf{V} \cdot \nabla \zeta + \varpi \frac{\partial \zeta}{\partial p} + \mathbf{k} \cdot \left(\nabla \varpi_{\mathrm{A}} \frac{\partial \mathbf{V}}{\partial p} \right) + (\zeta + f) \nabla \cdot \mathbf{V} = 0 \quad . \quad (1)$$

$$\frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T + \Gamma \varpi = 0 \quad . \quad . \quad . \quad . \quad (2)$$

$$\nabla \cdot \mathbf{V} + \frac{\partial \varpi}{\partial p} = 0 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$T + \frac{gp}{R} \frac{\partial h}{\partial p} = 0 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where $\mathbf{V}$ is the horizontal velocity, ζ the vorticity associated with $\mathbf{V}$, ϖ is dp/dt corresponding to vertical velocity in this system of co-ordinates, T is the temperature, p the pressure, h the height corresponding to a pressure p, Γ a measure of the stability of the atmosphere, usually taken to be approximately constant, f is the Coriolis parameter and $\mathbf{k}$ is a unit vertical vector.

Numerical predictions have been made in which the horizontal-velocity field has been obtained from the contour-height field through the geostrophic relation. The models which use the contour heights at two pressure levels as basic data have given predictions at a mid-tropospheric level and a level near the surface, assuming that the vertical-velocity profile at a geographical location is dependent only upon pressure; the dependence has usually been taken to be parabolic, zero near the surface and at some pressure level near the top of the troposphere. The vertical velocity is then characterized by a single parameter. (Sawyer and Bushby 1953; Thompson and Gates 1956). The computations have successfully predicted the motion of synoptic features and the computed vertical velocities at the mid-level of the troposphere agree in sign and often in magnitude with the vertical velocities that are inferred from the observations of weather and development. For example, predictions made using the Sawyer-Bushby model generally show ascending air in the regions of falling pressure and of rain. It is reasonable to suppose that the main features of the vertical-velocity profile are reasonably estimated by using the geostrophic wind as an approximation to the wind, for computations of advection and vorticity.

If we assume in addition to Eqs. (1) to (4) that

$$\mathbf{V} = \frac{g}{f}\mathbf{k}_{\wedge}\nabla h \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where g is the acceleration due to gravity, then we may eliminate the time derivatives from the system of equations, obtaining

$$\frac{\Gamma R}{gp}\nabla^2\varpi - \frac{f(f+\zeta)}{g}\frac{\partial^2\varpi}{\partial p^2} = \nabla^2\left\{\frac{g}{f}J\left(h, \frac{\partial h}{\partial p}\right)\right\} - \frac{\partial}{\partial p}J\left(h, \frac{g}{f}\nabla^2 h + f\right) \quad . \quad (6)$$

The so-called ' twisting term,' the fourth term of Eq. (1), involving the shear and the gradient of ϖ was omitted in deriving Eq. (6) because it is a relatively small term; the term involving $\varpi\,\partial^2\zeta/\partial p^2$ was similarly omitted. Eq. (6) gives ϖ in terms of the contour heights h of the pressure surfaces at a fixed time; it is not a prognostic equation. A formal solution cannot be obtained and the finite difference analogue of Eq. (6) must be solved to give an approximate solution.

Contour charts were constructed, from the radiosonde observations, at 100 mb intervals from 1,000 mb to 200 mb. The contour heights were obtained from each of these charts at the grid points of a square grid in a stereographic projection of grid length 200 mi at the North Pole. The total number of grid points at each level was 480, the points on and within an area bounded by 24 × 20 points. ϖ was taken to be zero at all the exterior grid points and this assumption will lead to errors at points near the boundary.

In forming the finite-difference equation, vertical derivatives were replaced by centred differences and the usual simple finite-difference scheme was used in the horizontal; allowance was made for the variation in map scale. The equation was solved by the Liebmann process, using 20 sweeps in the horizontal surfaces before moving to the next surface in the vertical. Experiment showed that when the field was taken to be zero everywhere at the start of the iterative process, 30 iterations of 20 sweeps in each horizontal were sufficient to give convergence.

3. Results

There is some difficulty in presenting the large number of numerical results in an easy visual form. They are represented in Figs. 1 and 2 by showing the vertical profiles of ϖ at each grid point on a square grid, at the geographical points to which they refer on a

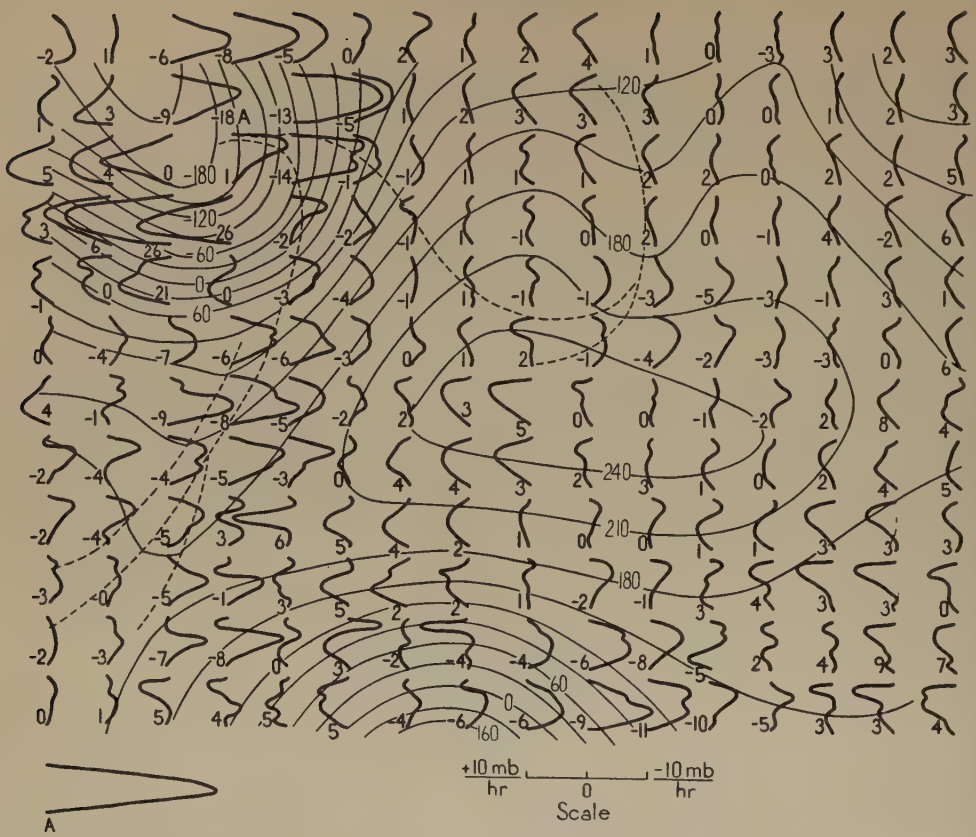

Figure 1. Profiles of ϖ for 0001 GMT 2 Dec. 1958 and the corresponding 1,000 mb contour chart (metres).

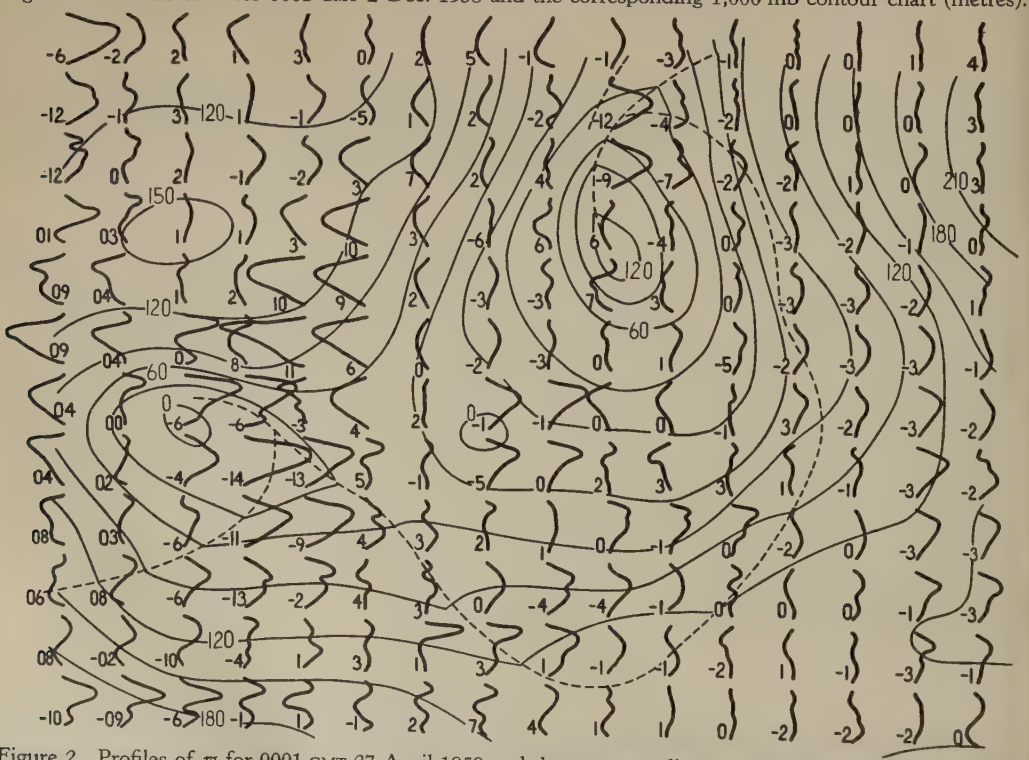

Figure 2. Profiles of ϖ for 0001 GMT 27 April 1959 and the corresponding 1,000 mb contour chart (metres).

Figures 1 and 2. The numbers indicate the corresponding values of ϖ_m obtained by using the Sawyer-Bushby model. 1,000 mb contours are given in continuous lines, heights being in metres, and the fronts are shown by pecked lines.

stereographic representation of the area. Although the equation for ϖ was solved at each level for a grid of 20 × 16 points, the results are presented over an inner area of 16 × 12 points, since the values obtained near the boundaries must differ from the true values owing to the artificial boundary conditions which have been assumed. In plotting the profiles, negative values of ϖ have been plotted to the right, and positive values to the left, of the origin, so that values to the right of the origin indicate ascending air. The synoptic chart at 1,000 mb for each case has been copied on the same diagram, but the outline of the land areas has been omitted, since its inclusion would have made it difficult to see the profiles. (Fig. 7 shows a geographical map of the area).

The analysis and subsequent evaluation of ϖ was made for two synoptic situations, 0001 GMT, 2 December 1958 and 27 April 1959. They were chosen as having distinct synoptic features. Perhaps the most striking feature of the profiles is their consistency in shape in that a feature of any particular profile is discernible in the nearby profiles, excepting, of course, in regions where there is a complete change in the vertical-velocity structure, the air descending at a set of grid points and ascending at a neighbouring set. The sign of ϖ is generally that which might be expected, upward motion in and ahead of low pressure systems and downward motion in their rear.

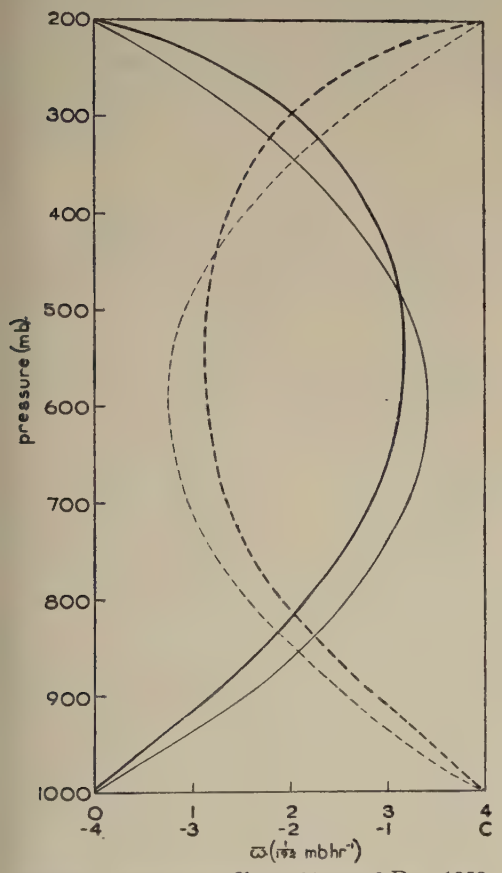

Figure 3. Mean ϖ profiles 0001 GMT 2 Dec. 1958.

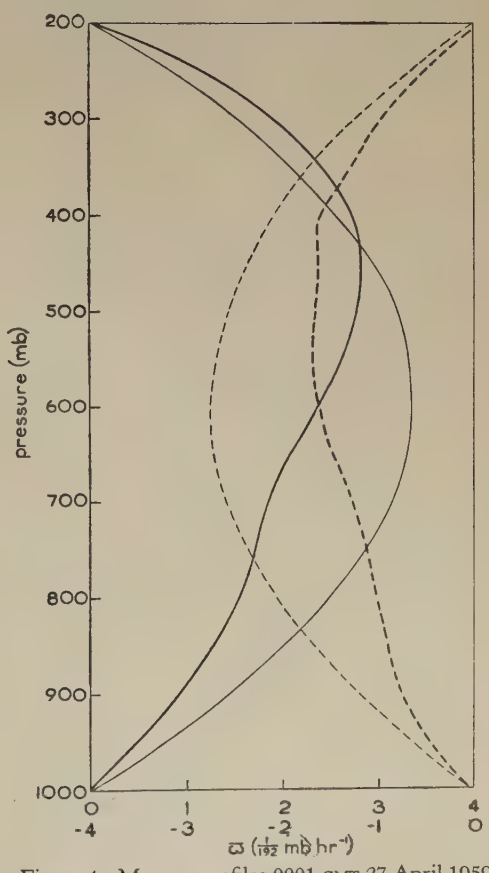

Figure 4. Mean ϖ profiles 0001 GMT 27 April 1959.

Figures 3 and 4.

- – – – – Mean upward motion
- – – – – Corresponding mean upward motion using Sawyer-Bushby model

} ϖ negative

——— Mean downward motion
——— Corresponding mean downward motion using Sawyer-Bushby model

} ϖ positive

From the general appearance of the profiles, especially in regions where the vertical motion is comparatively large, it seems that they are approximately parabolic in form. The mean profiles for upward and downward motion are given for both dates in Figs. 3 and 4. The corresponding mean parabolic profiles given by the Sawyer-Bushby model are also given. It appears in each case that the level of non-divergence is in the mean above the mid-tropospheric level, both for upward and downward motion and that the mean value of ϖ as computed using the Sawyer-Bushby model is generally overestimated.

(*a*) *2 December 1958*

The notable features on the synoptic chart for 0001 GMT, 2 December 1958 are the deep low in the north-west with the trough extending to the south and the anticyclone in the central area. The vertical velocities show a discontinuity almost coincident with the axis of the north-western trough, with rapidly ascending air ahead and rapidly descending air behind. The upward motion reaches a maximum at the point marked A, the profile of which is shown separately since it could not be entered into the diagram without loss of clarity; the value of ϖ at 600 mb is -28 mb hr^{-1}. Large negative values of ϖ are found over an extensive region in the forward side of the trough, with the maxima near the point of occlusion. In this area the profile of vertical velocity appears to be approximately parabolic in shape, with the level of non-divergence at the mid-tropospheric level. Fig. 5 shows the mean profile at the eight points in the north-west at which the

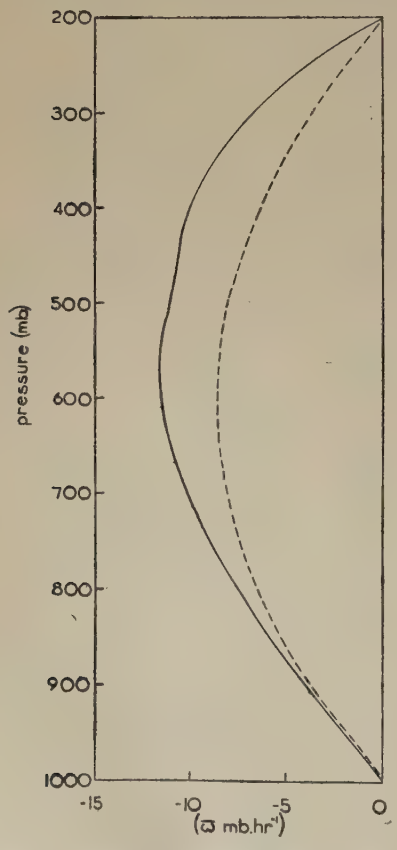

Figure 5. Mean ϖ profiles 0001 GMT 2 Dec. 1958.
———— Mean ϖ profiles ahead of trough
– – – – – Corresponding mean ϖ profile using Sawyer-Bushby model

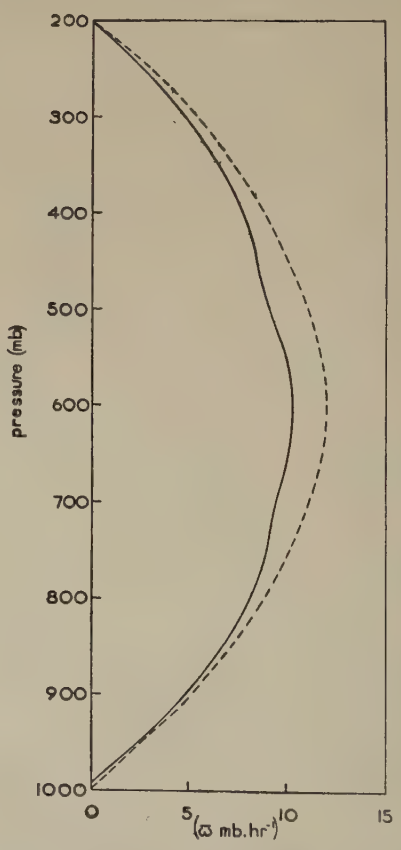

Figure 6. Mean ϖ profiles 0001 GMT 2 Dec. 1958.
———— Mean ϖ profile behind trough
– – – – – Corresponding mean ϖ profile using Sawyer-Bushby model

values of ϖ were largest in the vicinity of point A, the mean parabolic profile corresponding to the mean value of ϖ found at 600 mb and the corresponding mean parabolic profile computed using the Sawyer-Bushby model; the latter estimates are about 20 per cent less than the computed values, whereas in the mean, over all points, it seems that the Sawyer-Bushby model overestimates the vertical velocity. In the southern part of the trough the large upward motions are equally extensive in area, showing over a broad band of 300-400 mi. The parabolic form of the profile is broadly evident, but there are small-scale features superimposed.

The region of subsidence is sharply marked from that of upward motion, the change in the sign of the computed values occurring at adjacent grid points. The downward motion in this region is as large as the upward motion ahead of the trough and appears generally to be parabolic in form. Fig. 6 shows the mean profile at seven points behind the trough and the corresponding mean profile as computed using the Sawyer-Bushby model; in this area the latter are overestimates of the downward motion.

There is a considerable area of subsidence at the western end of the anticyclone, spreading back into the ridge ahead of the southern part of the trough. North of this region the level of non-divergence is in the upper troposphere and it gradually falls southward, where there are indications of upper positive and lower negative values of ϖ. To the north-west of the southern depression there is a well-marked region of upward motion, with the maximum at low levels near the depression, rising to the upper levels to the north-west. The south-eastern extension of the anticyclone shows the level of non-divergence to be in the upper levels.

(*b*) *27 April 1959*

There are two well-marked depressions and a small anticyclone on the synoptic chart. The northern depression has an associated secondary low. The vertical motions associated with the northern depression show that at only two grid points is ϖ markedly negative and they are near the tip of the occlusion. The parabolic form of these profiles is evident.

Figure 7. The area covered by the grid points of Figures 1 and 2.

There is a marked region of upward motion associated with the secondary low with the level of non-divergence in the upper troposphere at about 400 mb, whilst the region between these areas is generally one of downward motion; there is an abrupt change in the vertical motion at two neighbouring grid points. The area to the south-east of the cold front shows upward motion, about 400 mi ahead of the surface front.

The depression in the south-west of the chart shows some large values of ϖ in the region of the centre and to the east of it. Near the centre the profiles are approximately parabolic, but to the north the maximum values occur in the upper levels. South of the centre, in a weak trough ahead of the cold front, the profiles are not parabolic and there is evidence of downward vertical motion in the middle or lower layers. The anticyclone shows weak vertical motion with sharp subsidence in the ridge extending southeastwards. Near the point of occlusion of the western depression there is upward and downward motion at adjacent grid points, again showing the marked changes which may occur over a short distance.

For comparison purposes, the computed values of ϖ at 600 mb, obtained from the Sawyer-Bushby model assuming the profile of vertical velocity to be parabolic in form, have been entered as small figures at the grid points near the base of the profiles. On the whole these show a remarkable agreement with regard to sign with the computed maximum values obtained by using the more extended information at more levels, and very often with regard to actual value. Occasionally there is disagreement with regard to either or both and it is noticeable that in the vicinity of the secondary low of 27 April 1959, the Sawyer-Bushby computed values are quite small, whereas the more complete profiles show quite large upward motions. Near the boundaries such differences will occur because the boundary conditions are different in the two computations. In the Sawyer-Bushby computations the contour height is held constant at the boundary, whereas in the computations described here it is ϖ which is kept constant at the boundary. The grid points mentioned here are, however, in the middle of the computational region. The Sawyer-Bushby model uses information in the lower half of the troposphere and assumes that the upper half behaves in a similar fashion. The profiles show that in the lower troposphere the variation of ϖ with pressure is almost linear at these grid points, indicating no relative divergence. The Sawyer-Bushby model would assume that there was also no relative divergence in the upper half of the troposphere, whereas there was development there.

Conclusions

Firm conclusions cannot be based upon two computations; the ϖ profile in the region of an anticyclone or depression probably depends upon the age of the system. Nevertheless, we may draw certain tentative conclusions :

(a) The computed profiles are consistent with one another and with those that could be inferred from the synoptic observations.

(b) When there are large vertical motions they occur in mid-tropospheric levels. The upper and lower-boundary conditions have been taken to be $\varpi = 0$, but the introduction of small vertical velocities at these levels would have little effect upon the computed values when the vertical motions are large.

(c) In the mean the vertical-velocity profile is approximately parabolic. This is perhaps a less strong conclusion than (b) since it takes account of small maximum values of ϖ and may depend more upon the boundary conditions. Nevertheless, the conclusion is also true for large vertical velocities. The implication is that the assumption of a parabolic profile is reasonable in a model which has to assume a vertical profile depending upon a single parameter.

(d) There are large vertical motions near the point of occlusion.

(e) The vertical motion may change its character completely within a distance of 200 mi and the largest positive and negative values of ϖ may be found at adjacent grid points.

(f) The level of non-divergence is in the mean somewhat above the mid-pressure level, but at particular points may occur at any height. There are some points, particularly in regions of rather small vertical motion where there are two or more levels at which the divergence vanishes; when this is so the computations show that the corresponding levels may be discerned at neighbouring grid points.

Acknowledgment

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Numerical calculation of the displacements of a stratified airstream crossing a ridge of small height

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(Manuscript received 30 December 1959)

SUMMARY

Numerical computations have been made of the solutions of the equation

$$\frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^2 \psi}{\partial x^2} + l^2 \psi = 0$$

which describes the stream lines in two-dimensional flow of small amplitude in a stratified airstream. Solutions are computed for various variations of the parameter l as a function of height. ($l^2 = g\beta/U^2 - U^{-1} \partial^2 U/\partial z^2$ is function of the wind and temperature variations with height in the airstream). To render the mathematical problem determinate the value of l^2 is assumed to be constant above 16 km up to unlimited heights and by specifying the form of solution above 16 km consistent solutions are obtained for the troposphere little affected by the value of l^2 assumed at high levels. The solutions include lee-wave trains which diminish in amplitude downstream as well as those of constant amplitude. The method of solution results in the former appearing on the downstream side of the ridge only. (It is assumed that waves of constant amplitude can appear only on the downstream side).

Some typical displacements of the stream lines are illustrated in Figs. 3 to 10. They are similar to those computed by Queney (1947) and Scorer (1949) but it is demonstrated that lee waves may arise from variations of l^2 with height additional to those specified by these authors.

1. INTRODUCTION

Numerous theoretical studies have been made of the flow of a stably stratified airstream across an obstacle. In order to solve the equations of motion in a form which may be evaluated numerically, several important approximations and assumptions have been made. The more important simplifications which have commonly been made are :

(a) The flow is steady and non-turbulent

(b) Viscous forces are negligible

(c) Motion is adiabatic (condensation effects are neglected)

(d) Motion is two-dimensional (i.e., motion is over a ridge with no component parallel to it)

(e) The disturbance is of small amplitude (permitting the neglect of non-linear terms in the governing equations)

(f) Wind velocity and static stability are constant or have some specially simple variation with height in the basic airstream

(g) The horizontal scale is small enough to neglect the earth's rotation.

A consequence of (b), (f) and (g) is that occasions of separation of the flow from the ground are not treated.

The earlier work of Lyra (1943) and Queney (1947) was restricted to currents in which the static stability and wind speed in the basic current were the same at all levels. However, the treatment given by Scorer (1949) clearly shows that the vertical variation of both the stability and the wind speed can have very large effects upon the pattern of air motion over a barrier and, in particular, upon the amplitude and wavelength of lee waves. There have been a number of published discussions of the flow in airstreams in which either wind, stability, or both, vary with height, but in order to keep the mathematical analysis within bounds, the authors have been restricted to simple and often highly artificial air currents. Probably the most realistic formulation for which a solution has been obtained

is that used by Scorer (1954) and Wallington (1955) in which the atmosphere is divided into three layers each with a constant value of Scorer's parameter 'l^2.' Foldvik and Palm (1957; 1959) have also used a three-layer representation to illustrate the effects of the variations of stability in the upper troposphere and stratosphere.

The present study obtains solutions for the flow of air over a ridge in an airstream with arbitrarily specified variations of wind speed and temperature with height. In order to do this, use has been made of the electronic computer *Meteor*. The assumptions (a) to (h) above have been retained with the exception of (g), but it has been possible to improve upon the mathematical approximations used by Scorer (1949) and others when evaluating the displacement of the stream lines immediately over the disturbing ridge.

2. Theory

(a) *The basic equation*

If the two-dimensional flow over a ridge is represented by a stream function $\phi(x, z)$ and the stream function is resolved into harmonic components so that

$$\phi = \int_0^\infty \phi_k(k, z)\, e^{ikx}\, dk \quad . \quad . \quad . \quad . \quad (1)$$

then Scorer (Eq. 12, 1949) has shown that ϕ_k satisfies the equation

$$\frac{\partial^2 \phi_k}{\partial z^2} - \left(\frac{g}{c^2} + \beta\right)\frac{\partial \phi_k}{\partial z} + \left(\frac{g\beta}{U^2} - \frac{1}{U}\frac{\partial^2 U}{\partial z^2} - k^2\right)\phi_k = 0 \quad . \quad . \quad (2)$$

where z = vertical coordinate

$c^2 = \left(\frac{\gamma p}{\rho}\right)^{\frac{1}{2}}$ = speed of sound

$\beta = \theta^{-1}\,\partial\theta/\partial z$ (Here $\theta = \theta(z)$ is potential temperature in the undisturbed airstream)

$U = U(z)$ = wind speed in the undisturbed stream.

Noting that $g/c^2 + \beta = -\rho^{-1}\,\partial\rho/\partial z$ (where ρ = density) and writing $\psi_k = (\rho_0/\rho)^{\frac{1}{2}}\,\phi_k$ (where ρ_0 is the density for $z = 0$) reduces Eq. (2) to

$$\frac{\partial^2 \psi_k}{\partial z^2} + \left(\frac{g\beta}{U^2} - \frac{1}{U}\frac{\partial^2 U}{\partial z^2} - \frac{1}{2\rho}\frac{\partial^2 \rho}{\partial z^2} + \frac{1}{4\rho^2}\left(\frac{\partial \rho}{\partial z}\right)^2 - k^2\right)\psi_k = 0$$

The third and fourth terms in the bracket are of the order of $5 \times 10^{-3}\ \mathrm{km}^{-2}$ compared with values of the order of $1\ \mathrm{km}^{-2}$ for the first term. The former have therefore been ignored in the present work and the basic Eq. (2) becomes

$$\frac{\partial^2 \psi_k}{\partial z^2} + (l^2 - k^2)\,\psi_k = 0 \quad . \quad . \quad . \quad . \quad (3)$$

where $l^2 = g\beta/U^2 - U^{-1}\,\partial^2 U/\partial z^2$

(b) *The lower-boundary condition*

Eq. (3) needs to be solved subject to two boundary conditions. The boundary condition at the lower limit of integration, $z = 0$, is provided by assumption (e) of Section 1 which implies that the ground profile is a stream line.

For convenience it has been assumed that the ground profile has the simple form

$$H(x) = h\left(1 + \frac{x^2}{b^2}\right)^{-1} = \text{Real Part} \int_0^\infty hb \exp(-kb + i\,kx)\, dk \quad . \quad (4)$$

If the stream function is made zero on the profile $z = H$ where H is assumed small, then approximately

$$\phi + (\partial\phi/\partial z)\, H(x) = 0 \text{ for } z = 0$$

or

$$\psi_k = U_0\, hb \exp(-kb) \quad . \quad . \quad . \quad . \quad (5)$$

which provides the required boundary condition at $z = 0$.

Although solutions of (3) are obtained in terms of complex values of ψ_k, the real and imaginary parts of ψ_k will separately satisfy Eq. (3) and may be accepted as solutions of the problem if they satisfy the boundary conditions. Thus the real part of any solution of the form

$$\phi = (\rho_0/\rho)^{\frac{1}{2}} \int_0^\infty \psi_k \exp(i\,kx)\, dk$$

such that ψ_k satisfies Eq. (5) for $z = 0$, will be a solution for the stream function consistent with the ground profile given by (4). It is a solution of the problem posed.

(c) *The upper-boundary condition*

The solution of Eq. (3) is strictly indeterminate unless the values of l^2 are specified to indefinitely great heights, whereas our knowledge of the atmosphere is limited by the height attained by sounding balloons. However, it is physically unlikely that the computed flow in the lower troposphere will be greatly affected by the temperature and wind in the middle and upper stratosphere, and the procedure adopted in the present study has been to suppose that above a level of 16 km (a convenient upper limit to the integrations of Eq. (3)) the atmosphere is such that l is constant and equal to a specified value L. It will be shown that the choice of L has only a small effect at low levels.

The appropriate condition to be assumed at the upper limit of integration of Eq. (3) has recently been a matter of controversy (Scorer 1958; Palm 1958) even in the simple case of l independent of height. For reasons given by Corby and Sawyer (1958), it will be assumed here that the appropriate solution of Eq. (3) in the region above 16 km, where l is assumed constant and equal to L, is of the form

$$\left.\begin{aligned} \psi_k &= A \exp(i\,\nu_L\, z + i\,kx) \text{ where } \nu_L^2 = L^2 - k^2 \text{ for } k \leqslant L \\ &= A \exp(-\nu_L'\, z + i\,kx) \text{ where } \nu_L'^2 = k^2 - L^2 \text{ for } k \geqslant L \end{aligned}\right\} \quad . \quad (6)$$

ν_L and ν_L' are both taken as positive. (It will appear that these equations are consistent with the occurrence of wave motion on the lee side of the mountain ridge only).

Since the perturbation velocity u in the x-direction is equal to $\partial\phi/\partial z$, we can assume ψ_k and $\partial\psi_k/\partial z$ are continuous functions of z. Eq. (6) can therefore be used to provide boundary conditions at the level $z = z_1 = 16$ km which is used as the upper limit for the numerical solution of Eq. (3). These are

$$\left.\begin{aligned} \frac{\partial}{\partial z}(\psi_k) &= \quad i\,\nu_L\,\psi_k, \quad k \leqslant L \\ &= -\nu_L'\,\psi_k, \quad k \geqslant L \end{aligned}\right\} \quad . \quad . \quad . \quad (7)$$

(d) *Numerical solution of Eq. (3)*

A function $f(z, k)$ is specified satisfying Eq. (3) and satisfying Eq. (7) at $z = z_1$. Thus

$$\frac{\partial^2 f}{\partial z^2} + (l^2 - k^2) f = 0 \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and

$$\left.\begin{aligned} \frac{\partial f}{\partial z} &= -i\,\nu_L f, \quad z = z_1 \text{ and } k \leqslant L \\ &= -\nu_L' f, \quad z = z_1 \text{ and } k \geqslant L \end{aligned}\right\} \quad . \quad . \quad (9)$$

We also assume for convenience

$$f(z_1, k) = 1$$

Then ψ_k is a simple multiple of f which satisfies the lower-boundary condition (5); namely

$$\psi_k = U_0 hb \frac{f(z, k)}{f(0, k)} \exp(-kb)$$

and

$$\phi = U_0 hb \left(\frac{\rho_0}{\rho}\right)^{\frac{1}{2}} \int_0^\infty \frac{f(z, k)}{f(0, k)} \exp(-kb + ikx)\, dk \quad . \quad . \quad (10)$$

The displacement of the stream line at any level, z, is given by

$$\zeta = \text{Real part} \left(\frac{\rho}{\rho_0}\right)^{\frac{1}{2}} \frac{U_0}{U} hb \int_0^\infty \frac{f(z, k)}{f(0, k)} \exp(-kb + ikx)\, dk \quad . \quad . \quad (11)$$

(e) *Singularities and lee waves*

The integral in Eqs. (10) and (11) becomes indeterminate, and numerical integration becomes impossible if $f(0, k) = 0$ for any value of k. These singularities of the integrand correspond to the occurrence of lee waves as discussed by Scorer (1949). If, in order to make the integrals determinate, the integration is carried out along a line in the complex k-plane skirting the singularity of the integrand, the result will depend upon whether the contour of integration is completed on the positive or negative side of the real axis. The difference between the two values is $2\pi i \times$ (residue at the pole) or

$$2\pi i \frac{f(z, k_r)}{f'(0, k_r)} \exp(-k_r b + i k_r x) \text{ where } f'(z, k) = \frac{\partial}{\partial k}(f(z, k))$$

and this is a sinusoidal function of x representing wave motion set up by the ridge. Scorer (1949) has chosen the solution which places this sinusoidal disturbance on the lee side of the ridge, arguing that a finite disturbance extending indefinitely upstream would be inappropriate. For flow towards positive x, this choice corresponds to completing the integration in Eqs. (10) or (11) by passing on the negative imaginary side of the singularities. A similar choice is made by Lamb (1916) in a problem of surface waves where it is shown that the inclusion of very small dissipative forces produces a displacement of the singularity in the direction of imaginary k increasing and leaving the real axis along which integration is carried out towards the negative imaginary side of the singularity.

In the present work the integration will, therefore, be carried out along the contour shown in Fig. 1.

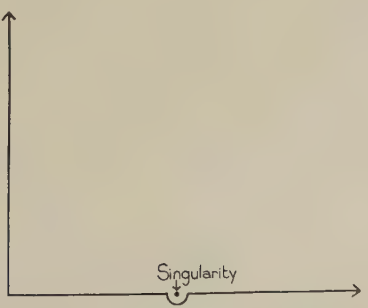


Figure 1. Contour used for integration with respect to k.

(*f*) *'Near-singularities'*

In the numerical solutions of Eq. (11) difficulty will arise not only when $f(0, k) = 0$ in the range of integration but also if it becomes very small. This occurs when $f(0, k) = 0$ for some complex value of k close to the real axis. With values of l^2 appropriate to atmospheric airstreams and the boundary condition (7) this commonly occurs, the imaginary part of k being of the order of $10^{-3} \times$ real part of k. It is mathematically convenient to deal with both the true- and near-singularities together, and it will be shown that both are related to the occurrence of lee waves.

(*g*) *Removal of singularities*

The integration of the expression (11) is made possible by first subtracting from the integrand a function which has a similar behaviour near the singular point. Such a function is :

$$\frac{f(z, k_r) \exp(-kb + i kx)}{f'(0, k_r)\,(k - k_r)} \quad . \quad . \quad . \quad . \quad (12)$$

where k_r is the value of k at the singular point. The expression (12) has a simple pole at $k = k_r$ with the same residue as the integrand in Eq. (11).

Eq. (11) is therefore rewritten in the form

$$\zeta = \text{Real part} \left(\frac{\rho_0}{\rho}\right)^{\frac{1}{4}} \frac{U_0}{U} hb \left\{ \int_0^\infty \left(\frac{f(z, k)}{f(0, k)} - \sum_r \frac{f(z, k_r)}{f'(0, k_r)\,(k - k_r)} \right) \exp(-kb + i kx)\, dk \right.$$
$$\left. + \sum_r \frac{f(z, k_r)}{f'(0, k_r)} \int_0^\infty \frac{\exp(-kb + i kx)}{k - k_r}\, dk \right\} \quad . \quad . \quad (13)$$

The summation is over all singularities and near-singularities. The integrand in the first integral is now free from singularities and numerical integration can be carried out. The integrals summed in the second term are dealt with as indicated in the following sub-section.

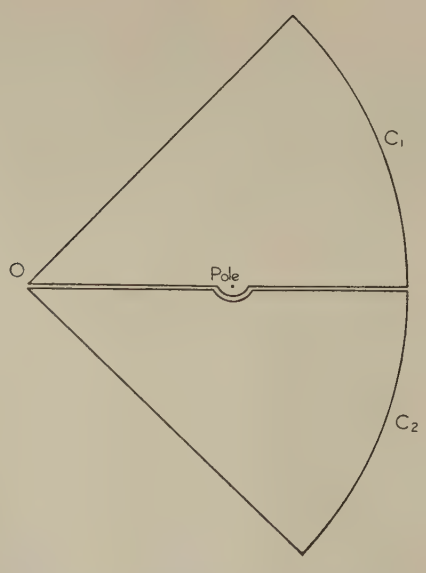


Figure 2. Contours used in evaluation of wave disturbance of stream lines; C_1 used for $x \geqslant 0$; C_2 used for $x < 0$.

(h) *Evaluation of the lee-wave terms*

The integral $\int_0^\infty \frac{\exp(-kb + ikx)}{k - k_r}\,dk$ is best evaluated by contour integration. The contour may be completed by the circle at infinity and a radius from the origin inclined at $+\pi/4$ to the real axis for $x \geqslant 0$ and $-\pi/4$ for $x < 0$. In Fig. 2 these contours are shown; C_1 applies for $x \geqslant 0$ and C_2 for $x < 0$.

The integrand has a pole at $k = k_r$. For a true singularity this lies on the real axis. For near-singularities it lies close to the real axis and it appears (although I am unable to demonstrate this rigorously) that, for the boundary conditions (9), it lies on the positive imaginary side. This has been confirmed by all the numerical evaluations which have been carried out. Thus, both for true-singularities (see sub-section (*e*)) and for near-singularities, the pole must be treated as lying within the contour C_1 and not within the contour C_2.

The integral along the circle at infinity vanishes and thus the required integral may be obtained from the integral along the radius and the residue at the pole. Thus for $x \geqslant 0$

$$\int_0^\infty \frac{\exp(-kb + ikx)}{k - k_r}\,dk = \int_{0\,(\text{radius})}^\infty \frac{\exp(-kb + ikx)}{k - k_r}\,dk + 2\pi i\,(\text{residue at } k = k_r)$$

where the integral on the r.h.s. is evaluated along the line $\arg k = \pi/4$. Hence for $x \geqslant 0$

$$\int_0^\infty \frac{\exp(-kb + ikx)}{k - k_r}\,dk = (1+i)\int_0^\infty \frac{\exp([-(b+x) + i(x-b)]\,y)}{(1+i)\,y - k_r}\,dy + 2\pi i \exp(-k_r b + i k_r x) \quad . \quad . \quad . \quad (14)$$

Similarly for $x < 0$

$$\int_0^\infty \frac{\exp(-kb + ikx)}{k - k_r}\,dk = (1+i)\int_0^\infty \frac{\exp([-(b-x) + i(x+b)]\,y)}{(1-i)\,y - k_r}\,dy \quad . \quad (15)$$

It is to be noted that the sinusoidal term $2\pi i \exp(-k_r b + i k_r x)$ appears only in the expression applying for $x \geqslant 0$ and that for true singularities (k_r real) it represents a simple harmonic wave train extending indefinitely. For near-singularities, k_r has a small positive imaginary part and the expression contains a factor $\exp(-k_{r2} x)$ where k_{r2} is the imaginary part of k_r. It therefore represents a wave train which decreases in amplitude for increasing x although in many of these cases studied, the decrease of amplitude is exceedingly slow.

(i) *The complete solution*

The complete solution for ζ, the vertical displacement of the stream lines at level z can thus be written

$$\zeta = \left(\frac{\rho_0}{\rho}\right)^{\frac{1}{2}} \frac{U_0}{U} hb\,\{\text{Real part } (X_1 + X_2 + X_3)\} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where

$$X_1 = \int_0^\infty \left\{\frac{f(z,k)}{f(0,k)} - \Sigma_r \frac{f(z,k_r)}{f'(0,k_r)\,(k-k_r)}\right\} \exp(-kb + ikx)\,dk \quad . \quad . \quad (15)$$

$$\left.\begin{aligned} X_2 &= \Sigma_r \frac{f(z,k_r)}{f'(0,k_r)}(1+i)\int_0^\infty \frac{\exp(-(b+x)\,y + i(x-b)\,y)}{(1+i)\,y - k_r}\,dy \\ X_3 &= 2\pi i\,\Sigma \frac{f(z,k_r)}{f(0,k_r)} \exp(-k_r b + i k_r x) \end{aligned}\right\} x \geqslant 0 \quad . \quad (17)$$

$$\left.\begin{aligned} X_2 &= \Sigma_\tau \frac{f(z, k_\tau)}{f'(0, k_\tau)} (1-i) \int_0^\infty \frac{\exp\{-(b-x)y + i(x+b)y\}}{(1-i)y - k_\tau} dy \\ X_3 &= 0 \end{aligned}\right\} x < 0 \quad . \quad (18)$$

3. Methods of numerical evaluation

A programme has been written for the electronic computer *Meteor* to evaluate the expression (15) for an airstream in which the values of θ, potential temperature, and wind U are specified arbitrarily at 1 km intervals of height from 0 to 16 km. For the purpose of numerical integration of Eq. (3) a height interval of 0·25 km is used and in order to evaluate l^2 at similar intervals interpolation is effected between the specified values of θ and U by carrying out central interpolation twice successively using the formula

$$\theta_{\frac{1}{2}} = \tfrac{1}{2}[\theta_0 + \theta_1 - \tfrac{1}{8}(\delta^2\theta_0 + \delta^2\theta_1) + \tfrac{3}{128}(\delta^4\theta_0 + \delta^4\theta_1)] \quad . \quad . \quad (19)$$

(a) *Evaluation of f and f'*

Numerical integration of Eq. (3) is carried out for 32 values of k in the range 0 to 5 km^{-1}. The simple finite-difference relation

$$f_{\tau-1} = -f_{\tau+1} + (2 - h^2(l_\tau^2 - k^2))f_\tau \quad . \quad . \quad . \quad (20)$$

was used, where h is the finite-difference interval of 0·25 km.

Integration is started at the 64th level corresponding to 16 km at which

$$f_{64} = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Conditions (9) were applied in finite-difference form to give the second condition

$$\left.\begin{aligned} f_{63} &= 1 - \frac{{\nu_L}^2}{2}h^2 - i\nu_L h \qquad k \leqslant L \\ &= 1 + {\nu_L}' h + \frac{{\nu_L'}^2}{2}h^2 \qquad k \geqslant L \end{aligned}\right\} \quad . \quad . \quad . \quad (22)$$

Values of $f'(0, k)$ were obtained by solving a similar finite-difference equation obtained by differentiating (20) with respect to k subject to boundary conditions obtained by differentiating (21) and (22). Thus

$$f_{\tau-1} = -f_{\tau+1} + \{2 - h^2(l_\tau^2 - k^2)\}f_\tau' + 2kh^2 f_\tau \quad . \quad . \quad (23)$$

is solved subject to

$$f_{64}' = 0$$

and

$$\begin{aligned} f_{63}' &= kh^2 + ikh/\nu_L, \qquad k \leqslant L \\ &= kh/\nu_L' + kh^2, \qquad k \geqslant L \end{aligned}$$

(b) *Location of singularities and near-singularities of* $f(z, k)/f(0, k)$

A preliminary search for singularities and near-singularities is made by evaluating $\left|\frac{f(0,k)}{f(z,k)}\right|^2$ for $z = 1$ km and k at intervals of 0·5 km^{-2} from 0 to 1 km^{-1} and for somewhat larger intervals to 5 km^{-1}. Minima of $\left|\frac{f(0,k)}{f(z,k)}\right|^2$ are used as starting points for an iterative Newton-Raphson procedure which locates the zeros of $g = f(0, k)/f(z, k)$. The iteration is based upon the recurrence relation

$$k_{s+1} = k_s - g_{k=k_s} \Big/ \left(\frac{\partial g}{\partial k}\right)_{k=k_s}$$

(c) *Evaluation of the integrals in Eqs. (16), (17) and (18)*

For large values of x the factor $\exp(i\,kx)$ in the integrand oscillates rapidly with k. The integrals are therefore evaluated as the product of a slowly varying function $A(k)$ with $\exp(i\,kx)$. Then the integral between two successive ordinates $k = k_1$, and $k = k_2$ used in the numerical integration is

$$\int_{k_1}^{k_2} A \exp(i\,kx)\,dk = \frac{2}{x}\left\{\sin\frac{(k_2-k_1)\,x}{2}\right\}\frac{A_1+A_2}{2}\exp\left(i\,x\,(k_1+k_2)/2\right)$$
$$-\frac{i}{x}\left\{\cos\frac{(k_2-k_1)\,x}{2}-\frac{2}{x\,(k_2-k_1)}\sin\frac{(k_2-k_1)\,x}{2}\right\}(A_2-A_1)\exp\left(\frac{i\,x\,(k_1+k_2)}{2}\right). \quad (24)$$

This relation has been used to evaluate the contribution of the interval between each pair of successive values of k to the integrals in Eqs. (16), (17) and (18). The intervals used in the integrations were 0·05 km^{-1} from 0 to 1 km^{-1}, 0·2 km^{-1} from 1 to 2 km^{-1} and 0·5 km^{-1} for $k > 2$ km^{-1}.

(d) *Mathematical errors*

The numerical integration of Eq. (16) for $z = 0$ was used to reconstruct the profile of the mountain height. Comparison with the simple expression (4) showed that the error was less than 1 per cent everywhere and there appears no reason to expect greater errors in the evaluation of the other integrals.

Truncation error is to be expected in the use of the finite-difference equations (20) and (23). In ignoring 4th-order differences and substituting

$$\partial^2 f/\partial z^2 = (f_{r+1} + f_{r-1} - 2\,f_r)/h^2$$

a proportionate error of the order of $l^2\,h^2/12$ is made. Typical values of l^2 are less than 1 km^{-2} over much of the range of integration although occasionally reaching 6 km^{-2}. Thus with $h = 0{\cdot}25$ km the error is generally less than 1 per cent although it may occasionally rise to about 3 per cent.

With these considerations in mind it is believed that the computed values of ζ represent the true solutions of Eq. (5) within a few per cent.

(e) *The programme on Meteor*

The programme as written for *Meteor* provides for the specification of U and θ at 1 km intervals from 0 to 16 km, and for the specification of the mountain half-width, b, and the value of L, the parameter specifying the stability of the fictitious atmosphere above 16 km. Values of the stream-line displacement, ζ, are calculated at horizontal intervals of 0·5 km or 2 km extending for 32 intervals on either side of the axis of the ridge. To calculate these values for 16 levels takes about 8 minutes.

4. Results

A typical pattern of stream lines computed by the method outlined in the preceding sections is illustrated in Fig. 3. This shows the vertical displacements of stream lines originating at 0, 1, 2 . . . . 15 km upstream (continuous lines). The unit in the vertical scale is the height of the ridge in the ground profile. The vertical scale is exaggerated in order to show the displacements clearly, and overlapping of the curves does not imply that the stream lines intersect. The vertical variation of wind, temperature and Scorer's parameter l^2 are shown to the left of the diagram.

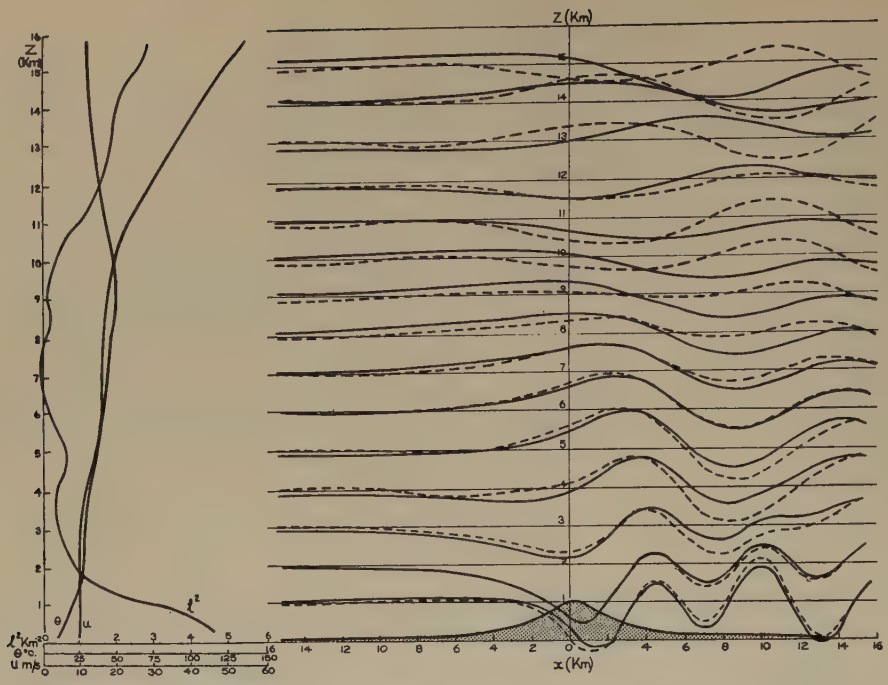


Figure 3. Displacement of the stream lines computed for a typical airstream. The left-hand section of the diagram shows the assumed potential temperature (θ), wind velocity (U) and Scorer's parameter, l^2, as function of height. The right-hand section of the diagram shows the computed displacements of the stream lines as functions of horizontal distance, x, at 1 km intervals in the vertical. The vertical displacement is plotted for each level on the same scale as the mountain profile at the bottom of the figure (mountain half-width, $b = 2$ km).

Upper-boundary value $L^2 = 6\ \text{km}^{-2}$ (continuous lines)
$L^2 = 0$ (broken lines)

In Fig. 3 and in subsequent Figures the vertical scale on which the displacements of the stream lines have been plotted is much greater than the scale used for vertical separation between the curves. This is necessary in order to show the features clearly, but has the consequence that stream lines appear to intersect. The theory which assumes small-amplitude disturbances would not apply if the ridge were 1 km high. If the diagrams are to be interpreted as applying to a ridge 0·25 km high, as would be appropriate, then to obtain a true picture of the stream lines the amplitude of all the curves must be reduced to one quarter of that shown.

Fig. 3 shows lee waves in the lower troposphere with a wavelength of about 5·7 km and a maximum amplitude comparable with the height of the ridge. The flow in the middle and upper troposphere is characterized by lee waves of somewhat smaller amplitude and longer wavelength, about 11 km. Disturbance of the stream lines comparable with the height of the ridge continue into the stratosphere. Another interesting feature is the strong downward flow over the crest of the ridge in the first few kilometers and the upward motion above the ridge at higher levels.

These features are typical of many of the calculations which have been done, but the amplitude and wavelength of the various features depend to a marked degree upon the characteristics of the airstream and the mountain width. These relations will be discussed in the following sub-sections.

(*a*) *Effect of the upper-boundary condition*

In the calculation resulting in the continuous lines Fig. 3, the value of L^2 appropriate to the atmosphere above 16 km was chosen to correspond to a highly stable stratosphere,

$L^2 = 6$. The calculation was repeated for other values of L^2 and the results for the extreme case of neutral static stability above 16 km, $L^2 = 0$ are illustrated by the dotted lines in Fig. 3. It is apparent that the main features of the computed flow pattern have not been altered, although some individual stream lines have been significantly displaced. Up to the 2 km level the differences are trivial.

The differences shown by these calculations probably illustrate a real sensitivity of the flow to the conditions above 16 km and cannot be overcome unless the wind and temperature at these levels are specified. (The sensitivity to conditions at high levels may be enhanced by the artificial restriction of the problem to two-dimensional flow over a ridge of infinite extent. In the real atmosphere where three-dimensional flow is induced by ridges of limited extent, dependance on high-level conditions may be somewhat less).

Although the choice of L^2 has some effect upon the solutions, it is reasonably clear that any general conclusions from the calculations regarding the flow in the troposphere will not be affected by the choice of L^2 if reasonably representative of stratospheric conditions.

(*b*) *Comparison with Scorer's approximate solution*

In order to avoid the need for evaluating complicated integrals of the form in Eqs. (16), (17) and (18), Scorer (1949) in calculating the stream lines in a two-layer atmosphere has adopted a device which is equivalent to replacing Eq. (16) by

$$X_1 = \frac{f(z, 0)}{f(0, 0)} \int_0^\infty \exp(-kb + i\,kx)\, dk \quad . \quad . \quad . \quad (25)$$

and putting $X_2 = 0$. He recognizes that this will give an unsatisfactory solution for small x, and his stream lines in this region are drawn by freehand interpolation.

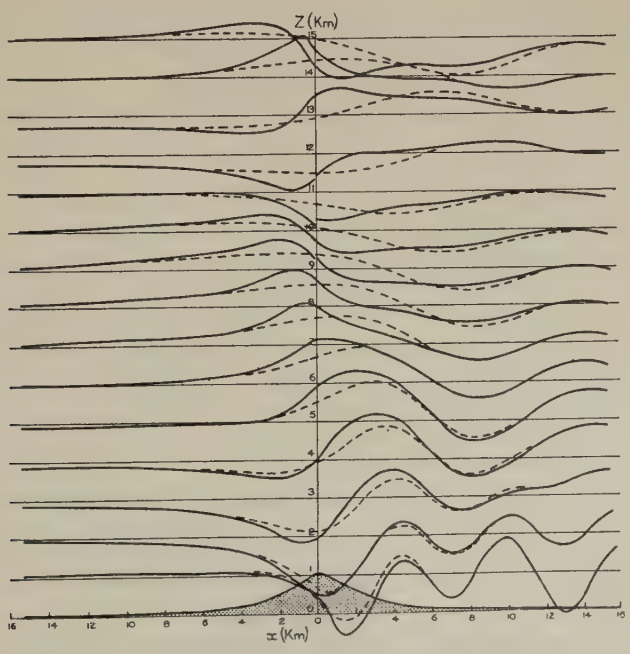


Figure 4. Displacements of stream lines computed for the same airstream as illustrated in Fig. 3, but using Scorer's approximation to the integral (continuous lines). Solution without this approximation is shown by broken lines for comparison ($b = 2$ km).

It was thought worth while to compare the solution using Eq. (25) with the complete solution of Eqs. (16) to (18). This is done in Fig. 4 for the same airstream and mountain width as illustrated in Fig. 3. The approximate solution is seen to be reasonably satisfactory in the lower troposphere although it over-emphasizes the descent in the immediate lee of the ridge. At higher levels the approximate solution shows a rather narrow region of ascent or descent above the ridge whereas in the complete solution the flow is smoother and the disturbance immediately above the ridge is of smaller amplitude.

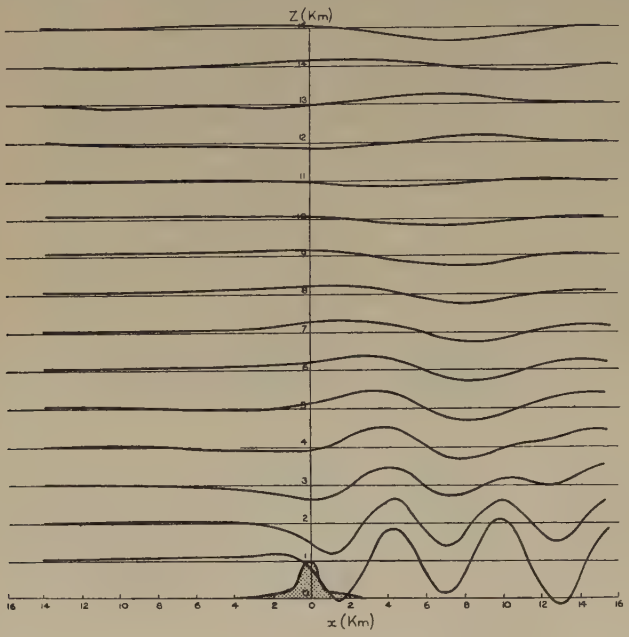


Figure 5. Displacements of stream lines computed for the same airstream as illustrated in Fig. 3 but for a narrow ridge ($b = 0{\cdot}5$ km).

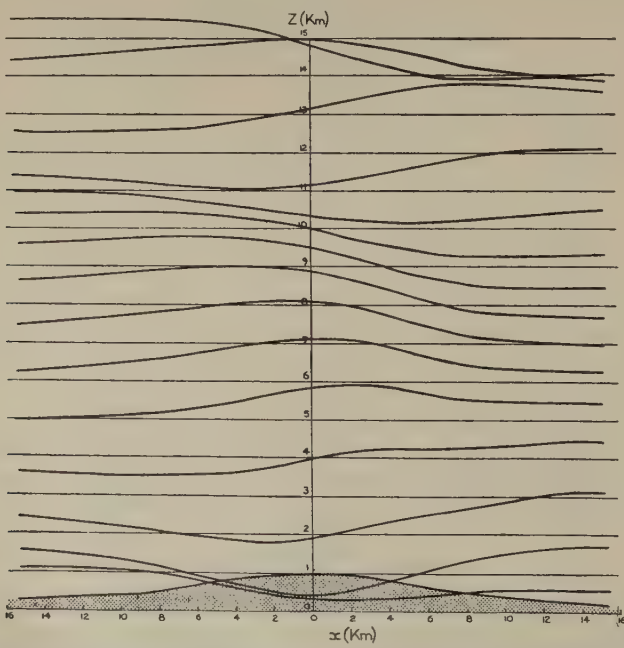


Figure 6. Displacement of stream lines computed for the same airstream as illustrated in Fig. 3 but for a broad ridge ($b = 8$ km).

(c) *Variation of the flow with the mountain width*

Corby and Wallington (1956) have demonstrated that as the mountain half-width, b, is increased the amplitude of the lee waves increases until a maximum is reached when $b = k^{-1}$; as b increases further the amplitude again decreases. The maximum for the shorter waves therefore occurs with a narrower mountain than for the longer waves.

This effect is shown in the present calculations which have been repeated for several different mountain widths. Two extreme cases are illustrated in Figs. 5 and 6 for the same airstream as used to obtain Fig. 3. In Fig. 5 the results are shown for a very narrow ridge, $b = 0{\cdot}5$ km; the main difference from Fig. 3 being that the disturbance in the upper troposphere and stratosphere is considerably less from the narrower mountain, and the longer-wavelength lee waves have smaller amplitude. The shorter-wavelength low-level waves are little affected.

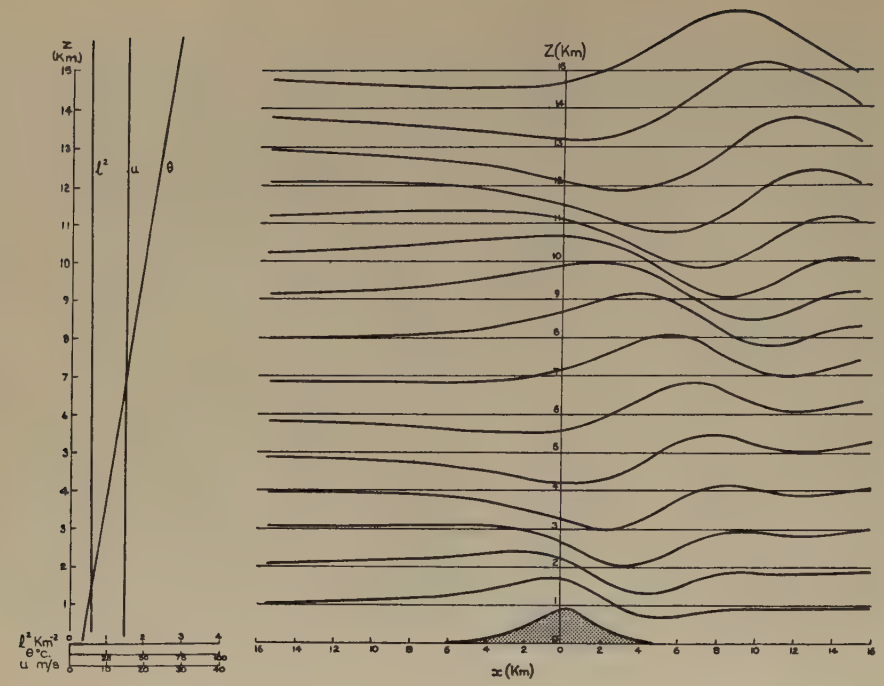
As the mountain half-width, b, is increased from 0·5 to 2 km the longer waves of the upper troposphere increase in amplitude. For $b = 4$ the short waves in the lower troposphere are considerably less than for $b = 2$. For $b = 8$ (illustrated in Fig. 6) both sets of waves are reduced to small amplitude and the disturbance of the flow lies mainly over the mountain ridge taking a form similar to that arising in airstreams in which l^2 is independent of height and which will be discussed in the following sub-section. It is noteworthy that in this airstream the air from 1 km up to about $4\frac{1}{2}$ km descends as it crosses the ridge but ascent appears again at higher levels. A substantial disturbance extends into the stratosphere with amplitude as great as the mountain height.

(d) *Some idealized airstreams*

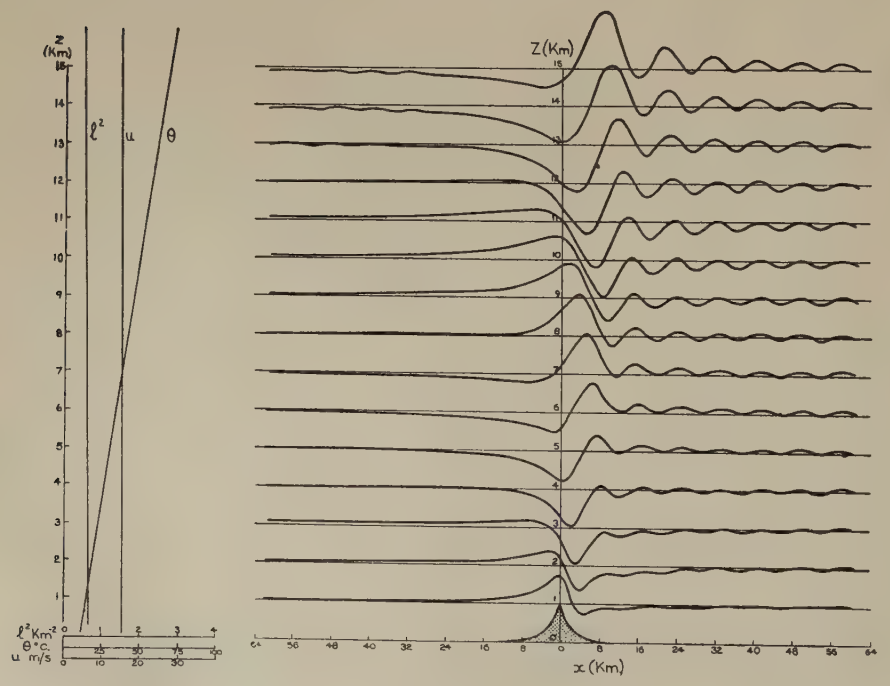
In order to illustrate the dependence of the computed stream lines upon the airstream characteristics, the calculations were made for values of U and θ representing the characteristics of some airstreams with simple variations of Scorer's parameter l^2 as a function of height.

Figs. 7 (a) and (b) give the results for an airstream in which ($l^2 \approx 0{\cdot}55$) is approximately independent of height. (Fig. 7 (a) has the same scale as Figs. 3-6, but in Fig. 7 (b) the horizontal scale has been contracted in order to show a greater extent of the flow). The flow pattern is similar to that calculated by Queney (1947) for a uniform airstream. In the immediate neighbourhood of the ridge the crest of the stream lines is tilted upstream and the vertical velocity undergoes an approximately sinusoidal variation with height. The stream-line pattern roughly repeats in the vertical with a wavelength of $2\pi/l$ although at high levels the flow is smoother and the strong downward flow on the lee slope is displaced about 2 km downstream when repeated around 10 km elevation. Fig. 7 (b) shows that a lee-wave train exists. The wavelength is $2\pi/l$, the amplitude is appreciably smaller than the mountain height, it increases upward and decays slowly downstream. The numerical technique of the present paper breaks down if the value of l^2 in the airstream is strictly constant and equal to L^2 (the value for the artificial upper layer). This is because $f(0, k)$ is discontinuous for values of k complex in the neighbourhood of $k = l$ and a singularity occurs at the discontinuity. With l not strictly constant the singularities are found at points not on the discontinuity of $f(0, k)$. There is no reason not to accept the solutions, and the waves in Fig. 7 (b) are presumably similar to those computed by Queney (1947) and Crapper (1959) for airstreams in which l^2 is strictly constant.

Scorer (1949) has demonstrated that an airstream which gives large values of the parameter l^2 in a layer of a kilometer or two near the ground and smaller values of l^2 aloft is particularly suitable for the development of lee waves. Stream lines in such an airstream are illustrated in Figs. 8 (a) and (b) (the profile of l^2 is on the left of Fig. 8 (a)). The occurrence of low-level lee waves of substantial amplitude is illustrated by the lowest two stream lines (1 and 2 km). The flow pattern at higher levels is little changed by the presence of the lower layer of large l^2 although the amplitude of the disturbance is slightly reduced.

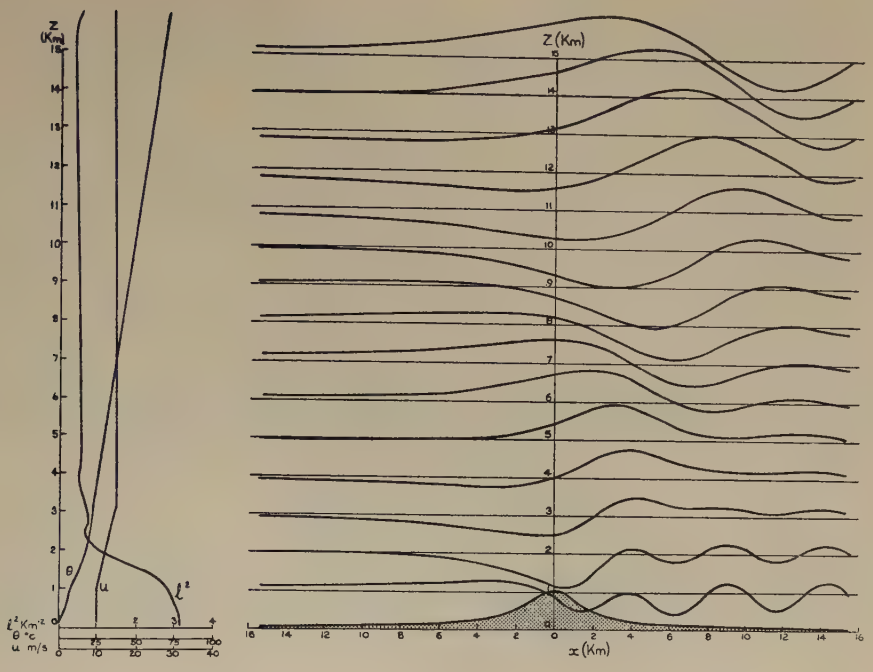


(a)

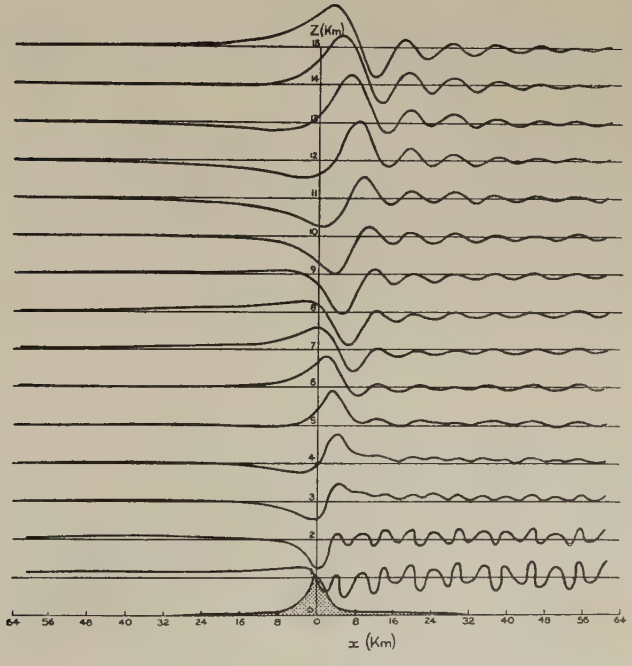


(b)

Figures 7 (a) and (b). Displacements of stream lines computed for an airstream for which l^2 is approximately independent of height. (For full explanation of diagram see Fig. 3). In Fig. 7 (b) the horizontal scale is one-quarter of that in Fig. 7 (a) – otherwise diagrams are similar.



(a)



(b)

Figures 8 (a) and (b). Displacements of stream lines computed for an idealized airstream with a layer of high l^2 values near the ground. (For further explanation see Figs. 3 and 7).

Damped lee waves can also arise if a layer of high stability (or large l^2) such as the stratosphere lies upon a deep layer with a substantially lower value of l^2. This may be demonstrated by considering an atmosphere consisting of two layers for which

$$l = l_1 \quad z > h$$

$$l = l_2 \quad z \leqslant h \quad \text{and} \quad L = l_1.$$

If we write $\nu_1^2 = l_1^2 - k^2$ and $\nu_2^2 = l_2^2 - k^2$ the solution of Eq. (8) which satisfies continuity of f and $\partial f/\partial z$ at $z = h$ is

$$f(z) = \exp(i\nu_1(z-h)), \quad z \geqslant h$$

$$= \frac{\nu_2 + \nu_1}{2\nu_2} \exp(i\nu_2(z-h)) + \frac{\nu_2 - \nu_1}{2\nu_2} \exp(-i\nu_2(z-h)), \quad z \leqslant h$$

As demonstrated in Section 2, damped waves will arise if $f(0) = 0$ for a value of k which has only a small imaginary part. If $l_1 \gg l_2$ such a value of k occurs near to the values k_0 for which $h\nu_2 = n\pi$. This may be demonstrated by solving the equation

$$\left[f(z)\right]_{k=k_0} + \left[\frac{\partial f(z)}{\partial k}\right]_{k=k_0} \Delta k = 0 \qquad . \qquad . \qquad . \qquad (26)$$

for Δk and showing that Δk is small ($\Delta k = k - k_0$). If $k < l_1$ we have $\nu_1 \ll \nu_2$; we also note that $\partial \nu_1/\partial k = -k/\nu_1$ and $\partial \nu_2/\partial k = -k/\nu_2$. Eq. (26) may then be written

$$1 + i\frac{\nu_1}{\nu_2^2} k_0 h \Delta k = 0$$

or

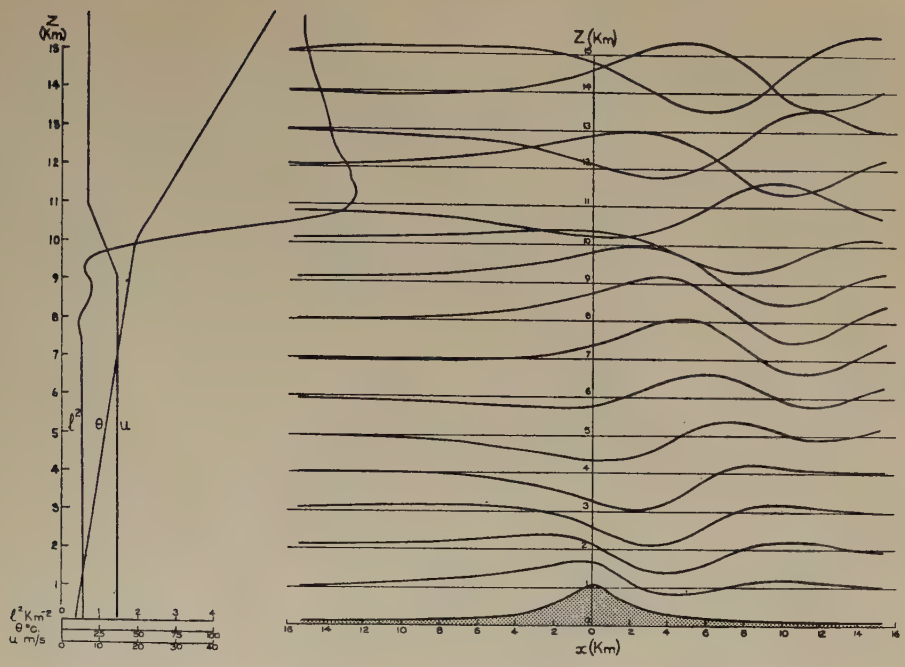
$$\Delta k = \frac{i\nu_2^2}{\nu_1 k_0 h}$$

which can be taken to be small under the conditions specified. The corresponding wave term given by Eq. (17) contains the factor $\exp(-\nu_2^2 x/\nu_1 k_0 h)$ which decreases exponentially with x.

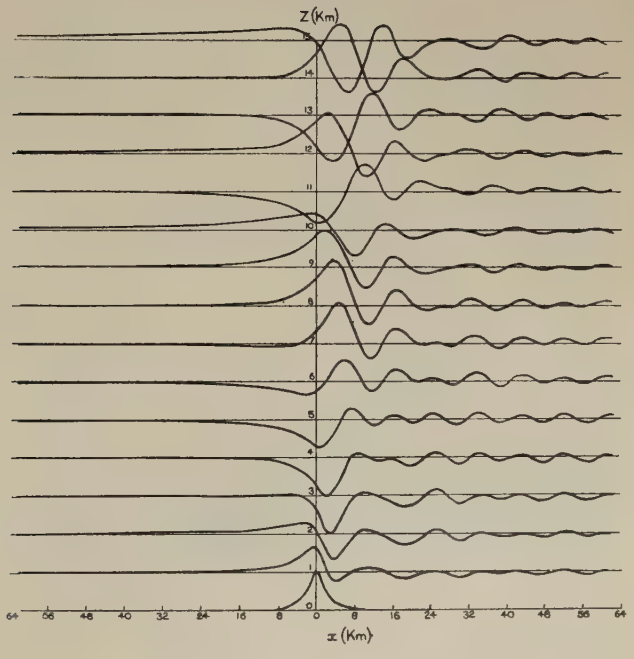
Calculation of the stream-line displacements has also been carried out for an airstream in which a high value of l^2 corresponding to that in the stratosphere occurs above a level of 10 km. Below this level the airstream has the same approximately constant value as used to obtain Fig. 7. The results of the calculations are illustrated in Figs. 9 (a) and (b). The main effect of the introduction of a stable stratosphere has been to introduce an additional rapidly decaying lee-wave train of longer wavelength than that arising without the stratosphere. This is shown most clearly in the stream line for the 2 km level where the first two oscillations are clearly defined.

(c) *Calculations for some natural airstreams*

The calculations of stream-line displacement were also carried through for values of θ and U interpolated from radiosonde and wind soundings to examine the solutions appropriate to natural airstreams. Eight examples were taken from those already studied by Wallington and Portnall (1958) and Corby (1957). One case was taken from the data on the ' Bishop wave ' in the report of the Sierra Wave Project (Holmboe and Klieforth 1957). The computed stream-line displacements in the latter example are illustrated in Figs. 10 (a) and (b) together with the stream lines deduced by Holmboe and Klieforth from aircraft and sailplane observations. In view of the uncertainty of the lapse rates (soundings some 120 mi distant were used) and the use of a very different ground profile, the agreement is reasonably satisfactory – particularly in regard to the longer apparent wavelength of the disturbance in the upper troposphere. The computed wavelength in the lower troposphere is 11·7 km compared with 14 to 18 km deduced from the flight observations.

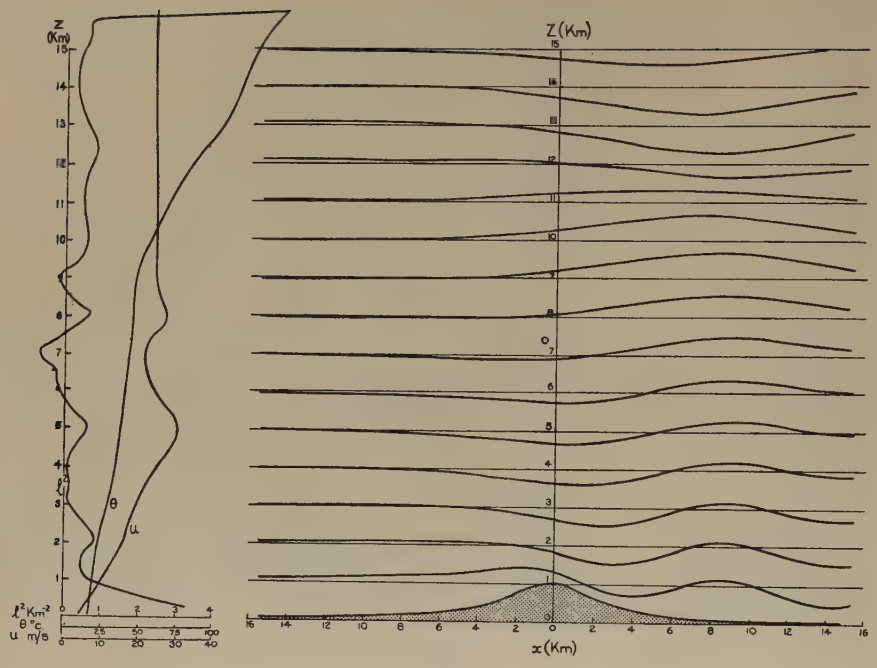


(a)

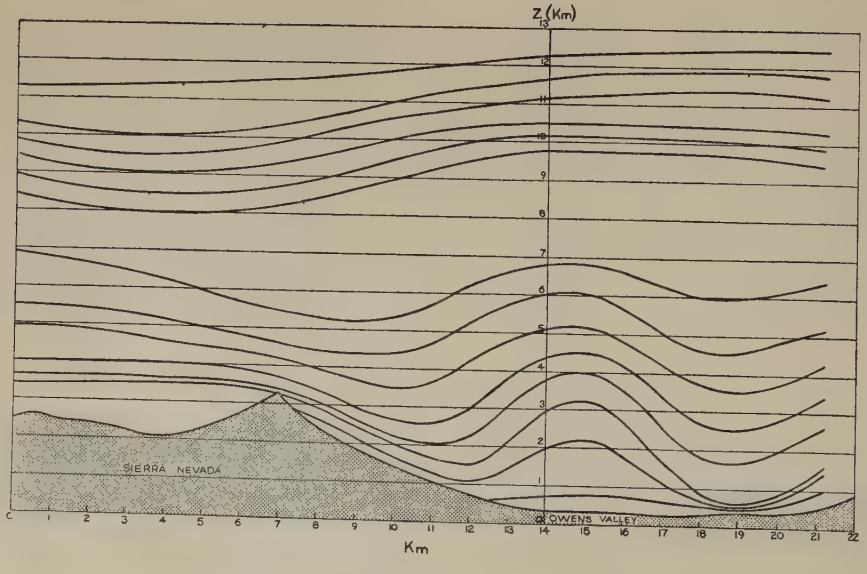


(b)

Figure 9. Displacements of stream lines computed for an idealized airstream with approximately constant value of l^2 in the troposphere and substantially larger value of l^2 in the stratosphere. (For further explanation see Figs. 3 and 7).



(a)



(b)

Figures 10 (a) and (b). Displacements of stream lines computed from data given by Sierra Wave Project for 18 December 1951. (a) computed stream-line displacements; (b) stream lines deduced from aircraft and sailplane observations.

The number of lee-wavelengths computed in the 14 natural airstreams examined varied from 1 to 3 per case. The individual flow patterns vary widely but have the general characteristics as illustrated in Figs. 3 to 10. The profiles of Scorer's parameter l^2 as a function of height have considerable irregularities, but this is not reflected in any irregularity of the stream lines. Lee waves in some cases can clearly be related to a substantial drop in l^2 from large values near the ground to lower values aloft, as suggested by Scorer. The amplitude of the disturbance then decreases with height. Other lee waves are not so clearly related to the l^2-profile and may have more of the character of waves induced by higher values of l^2 aloft as suggested in Section 4 (d). The amplitude of these waves is maintained or increased with height. There is even a tendency for increased amplitude in the stratosphere with rapid variation of phase with height.

In addition to the occurrence of lee waves, the computed flow in the immediate neighbourhood of the ridge line usually shows the features of Fig. 7 (a). However, the degree of tilt to the ridge in the stream lines and the height of the reversal in phase vary greatly from one case to another. Reversal of phase in the stream line occurs as low as 2 km in some cases of relatively light winds and stable lapse rate.

No simple relation appears to exist between the wind, temperature or l^2 profiles and the amplitude of the disturbance over, or to the lee of, the ridge. However, the wavelength of the shortest wavelength increases with the mean wind (here averaged from 0 to 10 km) and this is shown in Fig. 11 where the shortest wavelength computed is plotted as 1, the next longest as 2 and so on. This relation was pointed out by Corby (1957) from observational data and his regression line is indicated in Fig. 11.

Wallington and Portnall (1958) used a method very similar to the current one in order to compute the lee-wavelength. They determined the values of k which made the solutions of an equation equivalent to (3) vanish for $z = 0$. They tried various upper-boundary conditions and accepted values of k as representative of a true lee-wavelength only if they appeared for several different upper-boundary conditions. The lee-wavelengths determined in the present experiment agree with those obtained by Wallington within about 10 per cent. The differences can be attributed to the differences in interpolation of wind and temperature from the radiosonde soundings. The present method resolved the

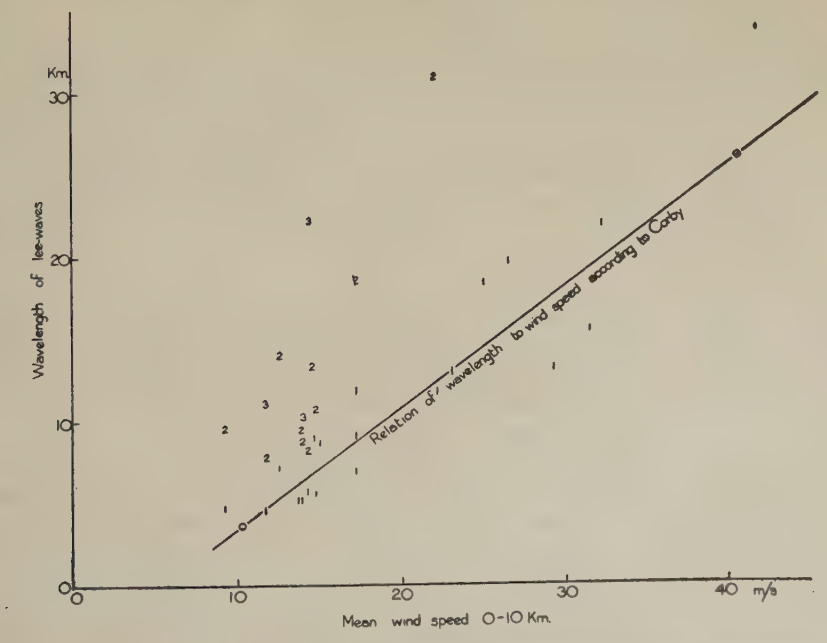


Figure 11. Relation between computed wavelength of lee waves and mean wind speed 0-10 km.

uncertainty which remained in some cases when examined by Wallington's method – the uncertainty arose if the value of k varied slightly but not greatly with the upper-boundary condition.

One case, that based on the Leuchars sounding for 14 GMT 20 November 1953, gave surprisingly large values for the lee-wave amplitude and for the disturbance of the stream lines over the ridge. The displacement of the stream line at 2 km was 6 times the height of the ridge causing the disturbance. It is not possible to point to any particular feature of the θ, U or l^2 profiles which accounts for the large amplitude. One feature is, however, a sharp minimum in wind speed at 2 km leading to negative values of l^2 at that level but negative values of l^2 have occurred in other cases studied without notable effect on the computed amplitude.

5. Conclusions

The present method of computation appears to give a satisfactory means of obtaining numerical solutions of the equations for steady two-dimensional flow of an airstream over a ridge when the solution is restricted to small amplitude disturbances. The treatment of the ' upper-boundary condition ' gives solutions which are consistent with the occurrence of lee waves downstream from an obstacle and with the upstream disturbance decaying to zero with distance upstream. (The direction of the wind current has, however, been assumed to be the same at all heights).

The computed displacements of the stream lines from the horizontal depend in a complicated fashion upon the variation of wind and potential temperature with height, but although there are wide variations between individual cases certain common features are usually found. These have already been recognized in previous papers on the subject and may be enumerated as follows :

(1) Lee waves occur on the downstream side of the ridge – waves of more than one wavelength may be superimposed.

(2) If two or more lee waves are present the shortest dominates the lower troposphere and the longer waves are conspicuous at higher levels.

(3) Down-slope flow on the lee side of the ridge is more intense than the up-slope flow on the windward side.

(4) The crest of the stream lines crossing the ridge is tilted upstream and the vertical displacement of the stream lines is a sinusoidal function of height.

(5) Upward displacement of the airstream over the ridge reverses with height in some cases at quite low levels.

Lee waves develop as predicted by Scorer (1949) when the value of Scorer's parameter l^2 is greater in a layer near the ground than aloft, and their amplitude then decreases with height. However, other distributions of l^2 as a function of height can also give rise to lee waves. Such lee waves maintain their amplitude up to the stratosphere, although they may decrease in amplitude downstream.

The result as stated in Section 4 and in the preceding conclusions describe the solutions of Eq. (3) but in applying them to the atmosphere the limitations of the mathematical formulation must be recalled. In particular, the restriction to two-dimensional flow makes possible the maintenance of substantial amplitude in the disturbances to high levels. Although similar flow probably takes place in the atmosphere over very long ridges transverse to the flow, a three-dimensional treatment of a more realistic ground profile would probably reduce the computed disturbance in the upper troposphere and stratosphere considerably.

The limitation of small amplitude must also be remembered and the numerical results illustrated in this paper cannot be regarded as applying to flow over ridges of greater height than 200 or 300 metres.

It is difficult to assess the importance of other restrictive assumptions (absence of turbulence, no separation from the ground, etc.) but it is believed that the computed results are illustrative of the type of flow which may occur in the atmosphere.

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Drop-size distributions from various types of rain

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(Manuscript received 11 November 1959; in revised form 5 February 1960)

Summary

Notable differences are found in the size distribution of raindrops reaching the ground from frontal clouds exhibiting a radar melting-band which is indicative of Bergeron-type rain; from small, warm shower clouds in which the rain is formed by coalescence; and from thunderstorms. These differences are illustrated by typical drop-size spectra and by plots of median-volume drop diameter against rainfall intensity for the three classes.

Introduction

Data on the size distribution of raindrops arriving at the ground have been published by many authors; the results appearing up to the end of 1956 have been reviewed by Mason (1957). The tendency has been to fit these data to simple empirical formulae, and in particular, to find relationships between any pair of the following parameters : median-volume drop diameter D_n, sum of the sixth powers of the drop diameters, $Z = \sum_N ND^6$, which is proportional to the intensity of the radar signal returned by the rain, the volume of rain-water per unit volume of air W, and the rainfall intensity p. While formulae such as those of Marshall and Palmer (1948) and of Best (1950) often represent quite well the relationship between the size distribution of raindrops in space and the rainfall intensity, considerable departures from these average or smoothed relationships may occur for individual samples taken over periods of only a few minutes. That the drop-size spectrum may change rapidly during the evolution of a shower is well known, and even in continuous, warm-frontal rain, short-period deviations from the Marshall-Palmer distribution are often observed – see, for example, Mason and Ramanadham (1953).

Moreover, since raindrops may form by different processes such as the melting of snowflakes, the melting of hailstones, the coalescence of water droplets, and because the drop-size spectrum may subsequently be modified by such factors as the coalescence of raindrops, the break-up of raindrops, accretion of cloud droplets by raindrops, evaporation of drops below cloud base, and the sorting of drops by wind shear, it may be expected that the characteristics of the spectra may be markedly different, according as they are associated with, say, widespread frontal or cyclonic rainfall, showers, and thunderstorms. Indeed, such differences may well be more interesting than similarities in smoothed or averaged data.

Marked differences between the characteristics of non-freezing, orographic rain and of freezing, non-orographic rain in Hawaii have been noted by Blanchard (1953); differences between drop-size spectra associated with a shower, with continuous warm-frontal rain, and with drizzle from low stratus are described by Kobayashi (1955), while the Z-p relationship obtained by Twomey (1953) for showers in Sydney, Australia, is considerably different from that which usually obtains for rain in middle latitudes of the Northern Hemisphere.

2. Present measurements of raindrop size

The object of this investigation was to obtain raindrop-size spectra in different types of rain with a view to correlating any distinctive features with the rainfall type, the probable release mechanism, and other characteristics of the rain as revealed mainly by the structure and evolution of the radar echoes.

The raindrops were sampled on the roof of the Huxley Building, Imperial College, London, using the well-known filter-paper technique and also, very recently, by an improved version of the automatic photoelectric raindrop spectrograph described by Mason and Ramanadham (1953).

Whatman No. 1 filter papers, 24 cm in diameter, and treated either with a solution of bromo-cresol green or rhodamine dye, were tautly clamped over the narrow rim of a metal hoop. These were exposed to the rain with the aid of a cover and a stop-watch, for any desired period of time. Because the calibration of the filter papers is affected by changes in humidity, the papers were stored before use, and also between exposure and measurement, in a desiccator over calcium chloride.

The recorded data included the filter-paper number, date, time of day, exposure time of paper, character and duration of rainfall, rainfall intensity, and notes on the general weather situation. This information was carefully matched to that obtained with a 3 cm radar operated on the roof of the same building. The radar has a peak power of 10 kw, a pulse length of 1 μ-sec, and a conical beam of width 4°. This beam sweeps in a vertical plane through the zenith on a chosen bearing and gives a RHI display of a vertical cross-section of the atmosphere. The filmed record of the radar screen also shows the date, time of day, bearing of the radar beam, the gain-setting of the radar receiver, and the rainfall intensity.

Raindrop-size frequencies for diameter intervals of 0·25 mm were read directly from the filter papers with the aid of a calibrated scale and a 10-channel counter. This scale was designed from a calibration curve based on measurements of the stains produced by water drops of known size impinging on the filter papers at their terminal velocity. Calculation of the size distribution of drops in space N_D (m^{-3} 0·25 mm^{-1}), from that on the filter paper, involves a knowledge of the effective filter-paper area (295 cm^2), time of exposure, drop count in each 0·25 mm size interval, and a representative terminal velocity for the drops within each size interval. As the terminal velocity changes rapidly with drop diameter for drops less than 0·25 mm diameter, the computed values of N_D for such drops are subject to error.

The intensity of the rainfall was computed from the size distribution of stains on the filter paper. For each 0·25 mm interval, an average drop mass was computed and multiplied by the drop number in that particular interval to give its contribution to the intensity. As pointed out by Blanchard (1953), this method of computing the rainfall intensity may not be highly accurate when the sample contains a statistically inadequate number of large drops which, nevertheless, represent a considerable fraction of the total water. For the very heavy showers, the greatest inaccuracy was introduced in timing the very short exposures of the filter papers which became saturated in about 5 sec. However, for the very-short-period samples taken in heavy-shower rains, it was considered to be more reliable than attempting to deduce instantaneous values from the rate-of-rainfall recorder.

3. Drop-size spectra from continuous warm-frontal rain

Raindrop samples were obtained on a large number of occasions with persistent, widespread rain falling from extensive layer-cloud systems associated with warm fronts and depressions. On those occasions the precipitation radar-echo characteristically exhibited a well-marked melting band indicating that the rain resulted principally from the melting of snowflakes. However, the rainfall was often far from steady and examination of the radar-echo on reduced gain often revealed that large fluctuations in the rainfall intensity and drop size were correlated with the presence of streaks, cells, or patches of heavier precipitation.

On occasions when such structural features were absent, and the radar signals both above and below the melting band were of nearly uniform intensity over periods of, at least, several minutes, the recorded drop-size distributions were typically of the type shown in Fig. 1. Here, the four distributions recorded with rainfall intensities ranging

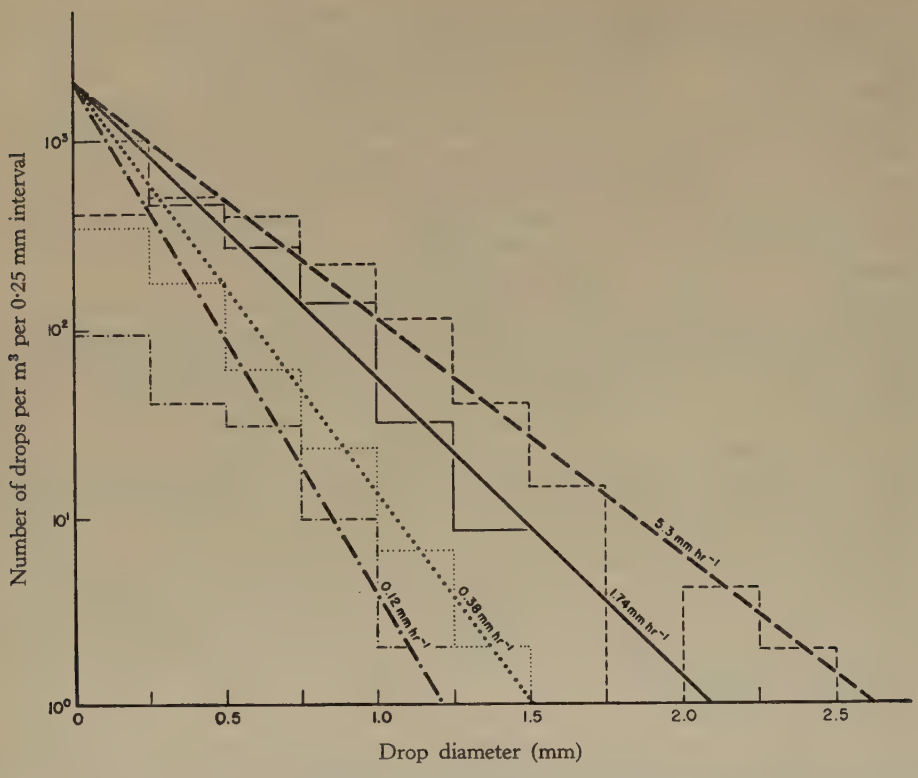

Figure 1. Drop-size spectra associated with warm-frontal cloud systems exhibiting marked radar melting-bands but otherwise little structure.

from 0·12 to 5·3 mm hr^{-1} are represented quite well by the formula proposed by Marshall and Palmer (1948) and indicated by the straight-line plots in Fig. 1. As Marshall and Palmer recognized, their formula over-estimates the number of small drops.

Temporary but marked departures from the Marshall-Palmer distribution are observed soon after the passage of localized areas of heavier precipitation through the radar beam. This is illustrated by the filter-paper samples depicted in Fig. 2 which are part of a sequence taken over a 90 min period during the passage of a warm front. During the sampling period a series of precipitation streamers or ' upper ' bands, of the type described in detail by Marshall (1953) and his collaborators, passed through the radar beam. The streamers, first detected at several thousand feet above the 0°C level, ultimately merged with, and caused local intensification of, the melting band. A few minutes later an increase in the rainfall rate was recorded at the ground.

Fig. 2 shows how the drop-size spectra changed as the rainfall intensity varied over the range 0·85 to 6·10 mm hr^{-1}. Distributions conforming fairly well to the Marshall-Palmer distribution (e.g., Nos. 479, 486, 489, 495), and associated with rather low rates of rainfall, are interspersed with others in which intensification of the rainfall is accompanied by narrower spectra with more large drops and fewer smaller drops than predicted by the Marshall-Palmer distribution. This suggests that, in localized regions characterized by intensification of the updraught, the precipitation elements grow appreciably by accretion, probably in both the ice and the liquid phases.

It is now apparent why, as stated in the Introduction, the Marshall-Palmer relation provides a good average representation of the drop-size distribution of warm-front precipitation but that considerable deviations occur over short periods of time. The observation by Mason and Ramanadham (1953) that, in short-period samples, there was often a

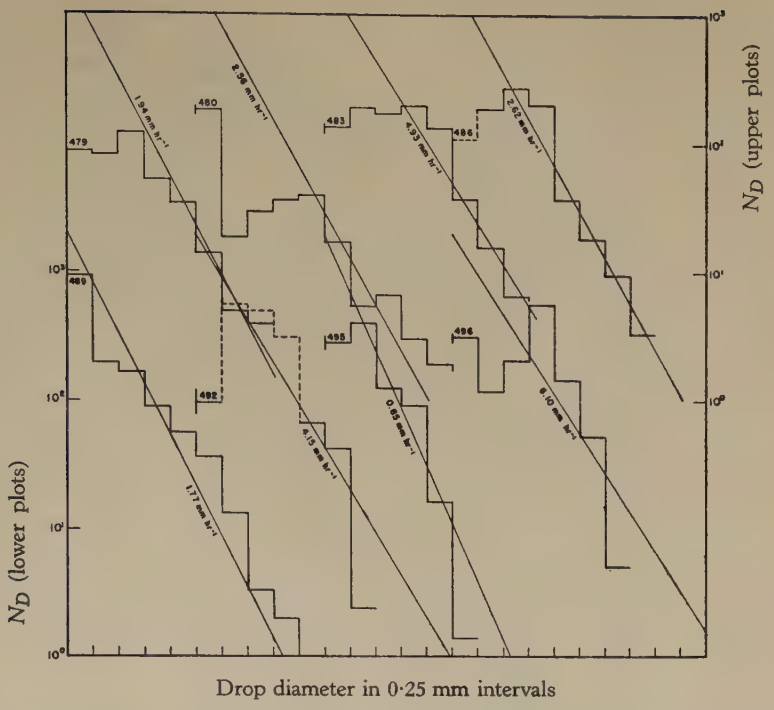

Figure 2. Size distributions of raindrops from frontal-cloud systems containing streamers and patches of heavier precipitation.

pronounced peak in the log $N_D - D$ curve at drop diameter of about 1·5 mm may well have been due to the passage of precipitation streamers or other localized regions of heavier precipitation.

According to Gunn and Marshall (1955), similar deviations might arise from sorting of the raindrops under the influence of wind shear, the largest drops reaching the ground nearest the generating source.

4. Raindrops from coalescence showers

The drop-size spectra associated with shower-type radar echoes which did not extend above the 0°C level have a distinctive shape and bear little or no resemblance to the Marshall-Palmer distribution. Feteris and Mason (1956) give details of 48 occasions in 1955 on which radar echoes developed in small shower clouds but did not extend above the − 2°C level; furthermore, the presence of an inversion or a stable layer usually did not allow the visual cloud to grow much above the maximum level of echo development. Similar observations have been made in subsequent years, and it is virtually certain that, in these cases, the rain was produced by the coalescence process.

Fig. 3 shows rain-drop-size spectra which are typical of those associated with coalescence showers. They form part of a sequence taken between 1313 and 1335 GMT on 28 June 1956. At this time the bases of the cumulus were 3,500 ft (8°C), and the radar echo of the shower under investigation reached a maximum height of 8,000 ft where the temperature was − 1°C. Confirmatory evidence was provided by the Meteorological Office Radar Station at East Hill which reported the maximum height of a nearby echo to be 8,500 ft between 1303 and 1307 hr, and by the Meteorological Research Flight which reported that cumulus tops in the general vicinity did not extend beyond above 8,000 ft between 1300 and 1400 hr.

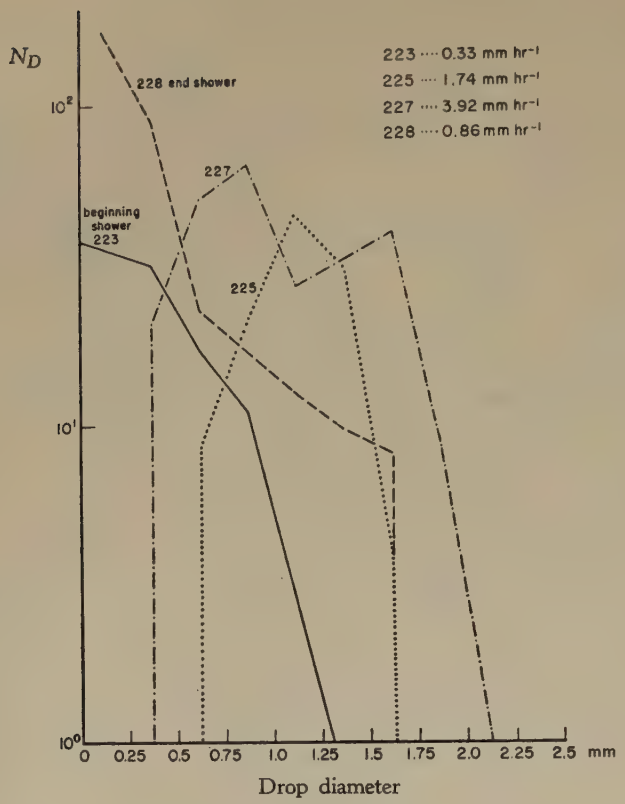

Figure 3. Drop-size spectra from small, warm shower clouds.

In the early stages of the shower, represented by sample No. 223 in Fig. 3, the rainfall was light and contained relatively large numbers of small drops. These were probably drops from the leading edge of the shower which partially evaporated during their fall, in the manner noticed by Atlas and Plank (1953). As the cloud moved overhead and the rainfall intensity increased to values of between 1 and 4 mm hr^{-1}, the spectra became rather narrow and exhibited sharp cut-offs at either end. It seems likely that small drops in the cloud were now being removed by accretion with larger drops and that the maximum size of the large drops was limited by the restricted depth of the cloud, the updraught velocity and the water content. In the dying stages of the shower, the proportion of small drops again increased.

5. Drop-size spectra from thunderstorm rain

The sizes of raindrops falling from thunderstorms have a broad spectrum, the heaviest rains containing drops of up to 6 mm diameter and large concentrations, approaching 10^4 m^{-3}, of small drops less than 0·25 mm in diameter. There are usually considerably higher concentrations of both the very small and the very large drops than would be predicted by the Marshall-Palmer distribution.

Drop-size spectra typical of thunderstorm rains are shown in Fig. 4. These are some of a series of samples taken as a thunderstorm passed over-head between 1223 and 1232 GMT on 2 July 1956. The storm was first detected on the radar at 1051 hr appearing as a strong columnar echo with top at 12,000 ft (— 8°C), two miles to the north-west of Imperial College. This continued to grow and the first cloud-to-ground lightning flash was observed at 1143 hr. The storm passed over the sampling station between 1223 and

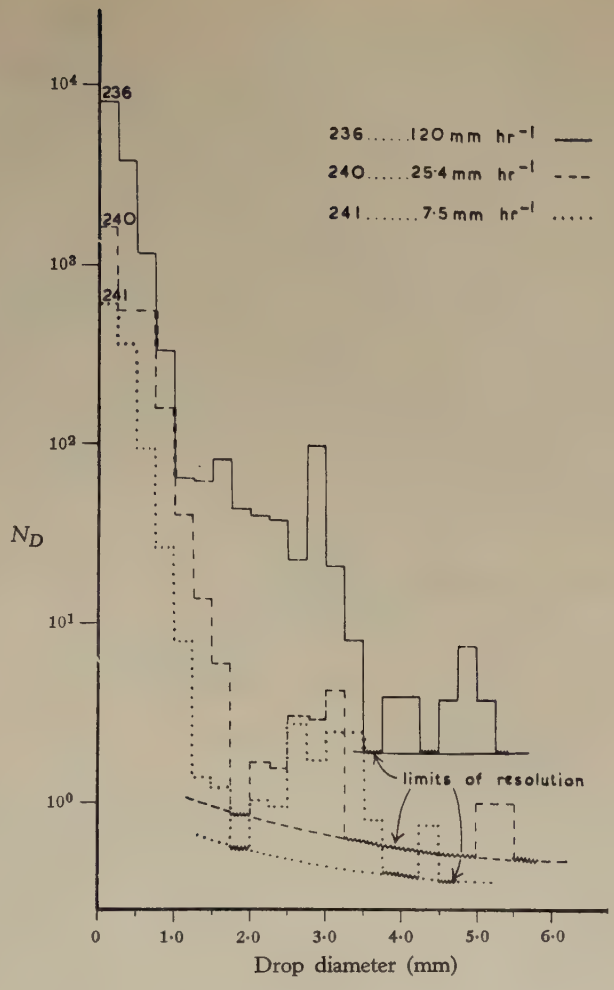

Figure 4. Size distributions of raindrops from thunderstorms.

1232 hr during which period the rainfall intensity varied from about 120 to 7 mm hr^{-1}, the former rate lasting for only a few seconds. Transparent hailstones, about 2 mm in diameter, fell between 1225 and 1226 hr.

The secondary peak in the drop-size spectrum, which occurs at drop diameters between 2·5 and 3 mm, may probably be attributed to the break-up of large raindrops during their fall.

The appearance, in very heavy rains containing very large drops, of proportionately larger numbers of small drops ($d < 0{\cdot}25$ mm) than are observed in all the other types of rain is also shown, but without comment, in Blanchard's (1953) paper.

6. Production of small drops by splashing

In the belief that these small drops may have resulted largely from splashing of the very large drops on the ground, a series of experiments was performed to determine the number of splash-drops produced by the impaction of water drops, 2·5 to 6 mm in diameter, falling at terminal velocity, on to some typical surfaces.

The drops, formed at the ends of glass tips, were allowed to fall freely for 100 ft down the well of the building and to impinge on horizontal targets consisting of dry and

wet paving stones, dry and wet soil, and a 3 cm depth of water. The number of splash-drops produced by, say, 100 drops of one size falling on a given target was determined by counting the number of stains produced on dyed filter papers placed at the same level as the target and up to radial distances of one metre from it. The procedure was repeated for different drop sizes and targets and the maximum heights attained by the splash drops were also measured. The results are summarized in Table 1. Splash production was highest with a wet paving stone covered by about 1 mm thickness of water*. The drop penetrated the surface of dry, lightly compacted soil without splashing, but wet soil, in which small shallow puddles developed, was a much more productive target. For all targets, the 5mm drops produced rather more splash-drops than the 6 mm drops; on average, each 5 mm drop produced 450 splashes from a wet paving stone and about 130 from a dry paving stone. Splash production decreased as the drop diameter fell below 5 mm and drops smaller than 2·5 mm diameter produced very few splashes. The fragments from the large drops rebounded to maximum heights of 30 to 60 cm. Most of the splash-drops were less than 0·5 mm diameter. The numbers of drops falling in the diameter range 0 to 0·5 mm in Fig. 4 may, in the light of the data in Table 1, be largely accounted for by the splashing of drops larger than 2·5 mm in diameter.

TABLE 1. AVERAGE NUMBER OF SPLASH DROPLETS PRODUCED BY DROPS OF DIFFERENT SIZES FALLING ON VARIOUS SURFACES

		Diameter of drop									
		6 mm		5·1 mm		4·45 mm		4·2 mm		3·45 mm	
Wet/Dry surface	Surface	Av. No. of splash drops	Max. Ht. of splash drops (cm)	No.	Ht.	No.	Ht.	No.	Ht.	No.	Ht.
Dry	Paving stone	104	60	133	35	95	35	98	35	57	30
Wet	Paving stone	343	65	449	55	320	55	222	55	92	45
Dry	Soil	–	–	1	30	–	–	–	–	–	–
Wet	Soil	–	–	65	45	66	45	63	45	–	–
	Water 3 cm deep	121	55	–	–	–	–	–	–	16	35

7. RELATIONSHIP BETWEEN MEDIAN-VOLUME DROP DIAMETER AND RAINFALL INTENSITY

The drop-size distribution may be characterized by any one of a number of parameters which may then be related to the rainfall intensity. We have chosen to plot, in Fig. 5, the median-volume drop diameter D_n against the rainfall intensity p to show up differences in the drop-size spectra for different types of rain.

The data for rain from warm-frontal cloud systems which exhibited a radar melting band conform fairly well to the relationship $D_n = 0{\cdot}92\, p^{0{\cdot}21}$ found by Atlas (1953) from the data of Marshall and Palmer and represented by the full straight line in Fig. 5. The data for coalescence showers show rather more scatter but may be represented by the dashed line having the same slope as the Marshall-Palmer relation but giving rather larger median-volume drop diameters for the same rainfall intensity. The thunderstorm data have a great deal of scatter, part of which is due to the statistically inadequate nature of the data for the higher rainfall intensities. Nevertheless there is a definite tendency for the median-volume drop diameter to be considerably larger than that for Bergeron-type rain.

* Similar results have been found by Gregory, Guthrie and Bunce (1959) when investigating the splash dispersal of fungus spores. They found that for drops of 5 mm diameter falling 2·9 m the number of splash droplets decreased from 1055 to 462 as they increased the water film from 0·1 mm to 1·0 mm. These figures include droplets of less than 100 μ diameter which we could not detect on our filter papers.

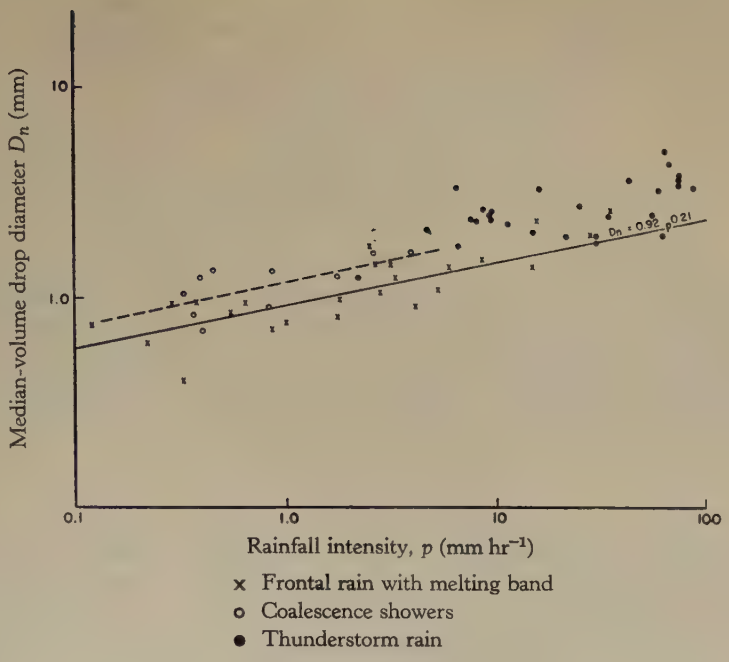

Figure 5. Plots of median-volume drop diameter versus rainfall intensity.

8. Conclusions

The observational data summarized in this paper point to the differences which exist between the size distributions of raindrops falling from different cloud systems and formed by different physical processes. The data also suggest that universal relationships between such parameters as the radar-echo intensity, $Z = \Sigma ND^6$, the liquid-water content, the median-volume drop diameter, and the rainfall intensity are not to be expected. Although more extensive investigations of this type may enable one to find empirical relationships which will hold, on average, for rain of a particular type, considerable deviations from any formula are to be expected on occasion because of such influences as wind shear, evaporation in the sub-cloud layer, and inhomogeneities in the precipitating cloud system.

Acknowledgments

One of us (J.B.A.) is indebted to the Director-General of the Meteorological Office for special leave of absence during which this research was undertaken. The authors are also indebted to all who assisted in the collection and processing of the data.

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Photolytic de-activation of silver iodide as an ice-forming nucleus

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(Manuscript received 17 August 1959)

SUMMARY

The rate at which the ice-nucleating ability of silver iodide decays when irradiated with the equivalent of strong sunlight has been determined by measuring the density of oriented ice crystals appearing on single crystals of silver iodide after various periods of exposure to a mercury-arc lamp. Wavelengths greater than 4,400 Å cause very little photolysis and decay of nucleation. The ice-crystal density decreases 10- to 20-fold after the silver iodide has been exposed to the equivalent of one hour's strong sunlight; this result is in fair agreement with the findings of the Australian workers where the activity of silver-iodide smoke falls by 1 to 2 orders of magnitude after one hour in the atmosphere in sunlight.

1. INTRODUCTION

That the ice-nucleating ability of a silver-iodide smoke decreases after irradiation by natural or artificial sunlight has been established in laboratory tests by Reynolds *et al.* (1951), Inn (1951), Vonnegut and Neubauer (1951), Birstein (1952) and Shimada *et al.* (1955). However, as Table 1 indicates, there is no general agreement as to the rate at which de-activation occurs for a given intensity of radiation. The decay rate appears to depend upon the manner in which the silver-iodide nuclei are produced as underlined by the results of Smith, Seely and Heffernan (1955) who released silver-iodide smoke and cadmium-zinc sulphide particles simultaneously from a ground site and traced these up to distances of 25 mi in an aircraft. The ratio of the concentration of AgI nuclei active at − 17°C in a mixing cloud chamber to that of the zinc-sulphide particles, determined at various times (distances) after release, was used to measure the deterioration of the silver iodide. Smoke produced by the vaporization of an acetone solution of silver iodide in a hydrogen flame retained only 1 per cent of its initial activity after 16 min exposure in daylight, whereas that from a kerosene burner deteriorated to the same degree only after 100 min. The rates of decay appeared not to be greatly dependent upon the altitude, cloud cover, or the humidity. Later experiments by Smith and Heffernan (1956), in which the smoke was released from a mountain top both by day and night, showed that decay proceeded much more slowly by night and, in the case of the kerosene burner, was indetectable after exposure to the atmosphere for more than 2 hr.

While most of the workers have found the nucleating ability to decay logarithmically with increasing time of exposure to light, the rates of decay appear to depend upon the manner and the temperature of nucleus production and the presence of impurities. Because of these uncertainties and the difficulty of sampling and testing smokes, we have studied large, irradiated, single crystals of silver iodide on which ice formation may be directly observed under the microscope and a direct, quantitative measure of the de-activation may be obtained by counting the number of nucleating centres for each area as a function of wavelength, intensity, and duration of the radiation.

2. EXPERIMENTAL METHOD

The silver-iodide specimens were prepared by the method described by Newman and Pashley (1955). Silver was evaporated on to freshly-cleaved mica in vacuo, the (111) faces of silver then being attacked, in darkness, by the vapour from a weak solution of iodine. This produces a very thin layer, 500-1,000 Å thick, in which the (0001) faces of hexagonal silver iodide and (111) faces of the cubic form are parallel to the surface, both forms giving an identical hexagonal array of iodine atoms upon which the basal plane of the ice lattice may form with a misfit of only 1·3 per cent.

TABLE 1. SUMMARY OF RESULTS ON PHOTOLYTIC DEACTIVATION OF SILVER IODIDE

Author	Method of production	Method of testing	Decay rate	Remarks
Reynolds *et al.* (1951)	Propane burner	Smoke stored in tank; irradiated with strong sunlight; tested in cold chamber at − 20°C	10^{-2}/24 hr	Considerable variation with cloud cover, time of day, particle size
Inn (1951) Birstein (1952)	Vaporized fused crystals from nichrome wire	Irradiated smoke with u.v. lamp of intensity comparable to sunlight on clear, summer afternoon. Tested in cold chamber at − 20°C	10^{-6} to 10^{-9}/hr	
Vonnegut and Neubauer (1951)	Ignited charcoal impregnated with AgI	As above	1/2 to 1/5/hr	
Shimada *et al.* (1955)	Vaporization at 600-700°C	Irradiated with strong sunlight and mercury-arc lamp. Tested in cold chamber at − 14 to − 16°C	2×10^{-3}/hr in strong sunlight	
Smith, Seely and Heffernan (1955)	Hydrogen generator Kerosene generator (1,440°C)	Smoke dispersed into atmosphere and sampled in airborne cold chamber at − 17°C	10^{-7}/hr 10^{-1}/hr	Activity decreased logarithmically with exposure time
Smith and Heffernan (1956)	Kerosene generator	As above	10^{-1} to 10^{-2}/hr negligible	By day By night
Smith, Heffernan and Thompson (1958)	Acetone solution of AgI atomized in combustion chamber of aircraft	As above	3×10^{-2}/hr 10^{-3}/2 hr negligible	By day By night

Such freshly-formed silver-iodide crystals were irradiated, for varying times, at a fixed distance from a 250 w Mazda high-pressure mercury-arc lamp, the interposition of suitable filters allowing the effect of different wavebands to be investigated. The ice-nucleating ability of the irradiated surface was studied by placing it in the apparatus described by Shaw and Mason (1955) in which ice crystals may be grown on a substrate under accurately determined conditions of temperature and supersaturation. The number of oriented ice crystals appearing on unit area of the surface, at $-17°C$ and at water saturation, was taken as a measure of its activity. Colour changes of the thin films, varying from yellow, through green and blue to brown, with increasing exposure, also indicated the progressive photolysis of the silver iodide to metallic silver.

3. Results

The specimen crystals were irradiated for periods varying from 5 min to 10 hr. By the successive use of filters cutting off at 4,500, 4,300 and 4,000 Å it was clearly established that wavelengths greater than 4,400 Å caused very little photolysis and decay of nucleation. This is in accord with the absorption band of silver iodide having a sharp cut-off at 4,300 Å. The use of additional filters cutting off at 3,600 Å and 3,800 Å indicated that the de-activation caused per unit of radiant energy was sensibly constant over the waveband, 3,500 to 4,400 Å. The intensity of the radiation between 3,000 Å and 4,400 Å incident on the specimens from the mercury arc was one-sixth of that which is received on a horizontal surface on Mt. Wilson (latitude 34° 11′ altitude 5,700 ft) during mid-summer with zenith sun and cloudless sky. For this reason, the exposure times given in Fig. 1 have been divided by six to give the equivalent exposures in strong sunshine.

During the first 5 min of irradiation by the lamp, the density of oriented ice crystals, (see Fig. 2, Plate III), increased about ten-fold but, thereafter, during exposures of up to 3 hr, the density fell quite rapidly, all parts of the silver-iodide surface deteriorating at about the same rate. For longer exposures still, some areas became completely inactive (see Fig. 3) while others showed little change. Further deterioration now occurred, mainly by extension of the inactive areas, and, under these circumstances, the activity was estimated by averaging over the whole surface.

For radiation doses equivalent to up to one half-hour's exposure in strong sunlight, the nucleating activity of the silver iodide decreased roughly logarithmically with a time constant of 20 min, as shown in Fig. 1, and thereafter decayed rather more slowly. During

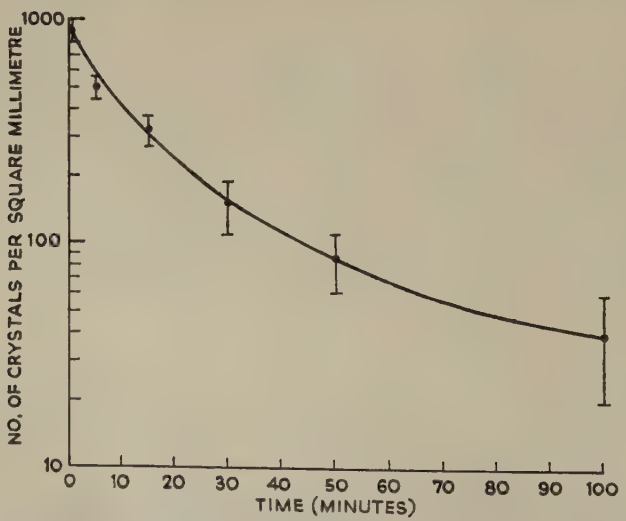

Figure 1. The density of oriented ice crystals growing on the surface of a silver-iodide crystal as a function of its time of exposure to the equivalent of strong sunlight.

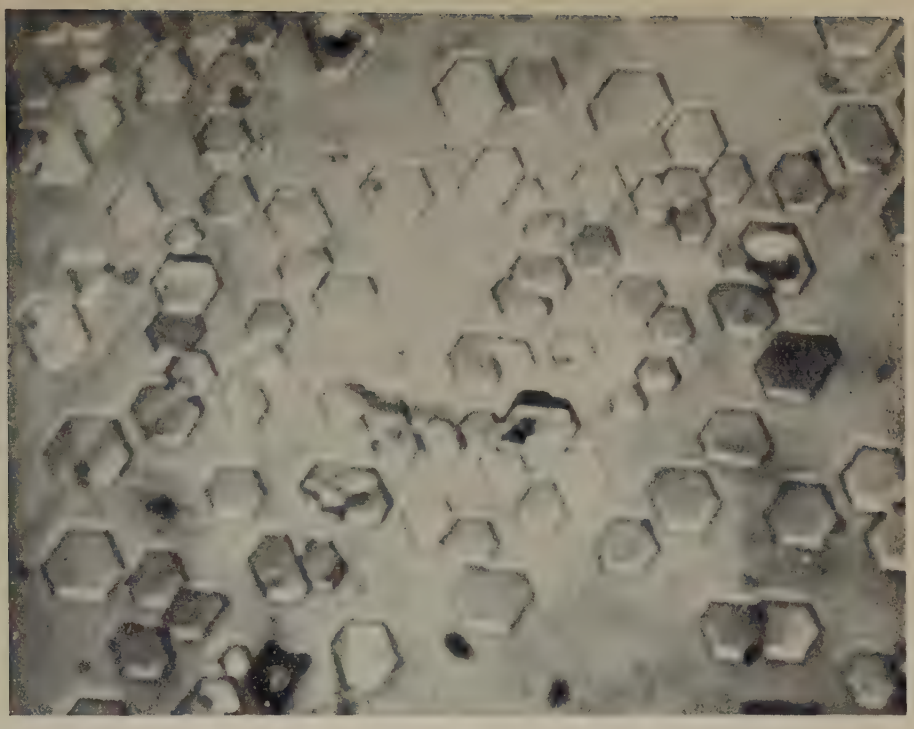

Figure 2. A dense deposit of oriented ice crystals growing on the basal plane of a silver-iodide crystal before exposure to ultra-violet radiation (× 350).

Figure 3. The density of oriented ice crystals is greatly reduced after 60 min exposure to the equivalent of strong sunlight (× 350).

the first hour's exposure the ice-crystal density fell to about 1/13th of the maximum value. This result is in reasonable agreement with decay factors of 10^{-1}-10^{-2}/hr obtained by the Australian workers (see Table 1) for smokes produced by a kerosene generator.

4. Effect of impurities

It is well known that the presence of trace impurities, especially sulphur, causes, in otherwise pure silver halides, a marked acceleration of photolysis. Specks of silver sulphide (Ag_2S), formed preferentially at the edges of growth steps emanating from dislocations on the surface of silver-iodide and silver-bromide crystals, form traps for the Ag^+ ions and free electrons released by photo-ionization of the halides, and facilitate their recombination to produce atoms of metallic silver.

Attempts to observe the effects of deliberate contamination of the silver iodide with H_2S failed because Ag_2S was formed accidentally by the sulphur present in the London atmosphere and because Ag_2S is also an efficient ice-nucleating substance at temperatures below — 8°C (Mason and Van den Heuvel 1959). In effect, all the silver-iodide crystals tested were probably sulphur-sensitized.

5. Conclusions

The ice-nucleating ability of single crystals of silver iodide was found to decrease by a factor of 10 to 20 after irradiation by the equivalent of one hour's strong sunshine. The de-activation was caused by the photolysis of the surface and its reduction to metallic silver by the ultra-violet component of the radiation with a sharp cut-off at about 4,300 Å. This rate of decay is in fair agreement with measurements made on silver-iodide smokes which were not produced in a high-temperature, reducing atmosphere such as is provided by a hydrogen generator. Heating to high temperature in a hydrogen flame would cause partial dissociation of the silver iodide followed by a migration of the silver ions to the many dislocations produced by the rapid heating and quenching. After irradiation, the silver ions will combine with electrons released by photo-ionization to form atoms of metallic silver. This process will be accelerated in the presence of traces of sulphur in the fuel and by the strong reducing effect of the hydrogen atmosphere, so that nuclei of metallic silver will be rapidly formed on the silver-iodide surface. The initial marked increase in activity brought about by the first 5 min irradiation is not understood; it may have been due to the removal of impurities from the surface.

Our results, confirming those of the Australian workers on the deterioration of silver-iodide smokes in the free atmosphere, suggest that nuclei released for rain-making purposes will lose much of their potency if they have to spend several minutes in bright sunshine before entering cloud. But since our measurements show that solar radiation in the waveband 3,000 Å to 4,400 Å is reduced about 5-fold by an overcast Sc. deck, the presence of an extensive deep-cloud system should afford considerable protection.

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Spectroscopic observations of the water-vapour content of the stratosphere

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(Manuscript received 6 October 1959; in revised form 5 January 1960)

SUMMARY

The water-vapour content of the stratosphere has been deduced from records of the solar spectrum observed from a Canberra aircraft at altitudes up to 48,000 ft. The spectral region 3,790-3,860 cm^{-1} in the 2·7 μ band has been separated into four groups of absorption lines whose rates of growth have been studied. Measurements in an altitude chamber of the absorption of very small paths of water vapour have been used to interpret the aircraft measurements in terms of absolute values of (pressure × quantity of water vapour)$^{\frac{1}{2}}$. Spectroscopic observations have been compared with measurements made by the frost-point hygrometer. The overall results show that the water-vapour content in a vertical column above 45,000 ft over southern England is about 4·5 μ, corresponding to a mixing ratio of 3×10^{-6}, if this is assumed constant above the height of observation.

1. INTRODUCTION

A knowledge of the water-vapour content of the stratosphere is of importance in three very different meteorological fields. Firstly, water-vapour mixing-ratio is a useful parameter in tracing the movements of air in studies of the general circulation; secondly, water vapour absorbs and emits infra-red radiation and makes a substantial contribution to the energy budget of the atmosphere; and thirdly, at levels above about 60 km water-vapour molecules take part in photochemical reactions, and probably provide the major source of atomic hydrogen in the ionosphere.

The first measurement of the humidity of the stratosphere over England was reported by Dobson, Brewer and Cwilong (1946). They used a frost-point hygrometer mounted on an aircraft and found the air in the stratosphere to be very dry compared with any air normally found at lower levels. The frost point which they measured of about − 80°C corresponds to a water vapour mixing-ratio of about 2×10^{-6} g of H_2O g^{-1} of air and to a relative humidity in the stratosphere of only 2 or 3 per cent. A large number of observations reported since 1946 by Bannon, Frith and Shellard (1952), and Murgatroyd, Goldsmith and Hollings (1955) confirm the early results. The frost point at heights between 45,000 ft and 50,000 ft – the maximum altitude at which observations have been made – appears to be remarkably invariable and lies between − 80°C and − 85°C. Further observations at these heights recently reported at latitudes from 68°N over Iceland to 33°N over North Africa show the same features (Helliwell 1958). The only direct measurements of humidity in the stratosphere over the American continent have been made by Barrett, Herndon and Howard (1950) using a balloon-borne automatic frost-point hygrometer. They reported values of frost point in the stratosphere 10°C to 20°C higher than those found over England.

Observations of the absorption of solar radiation by water vapour in the infra-red have been described by Yarnell and Goody (1952). Their observations are consistent with the frost-point measurements, although, since they were made at heights below the tropopause, they are rather insensitive to absorption by water vapour in the stratosphere. Observations of the solar spectrum have also been reported by Murcray, Brooks, Murcray and Shaw (1958) who used a balloon-borne instrument to investigate the near infra-red bands of water vapour at 1·4 μ and 1·9 μ. These bands are too weak for measurable absorption to be observed in the stratosphere with the low-resolution instrument they employed.

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In this paper results will be presented of a number of observations from a high-altitude aircraft of the absorption by atmospheric water vapour of solar energy at wavelengths near 2·7 μ. Comparison has been made between spectroscopic observations and measurements with the Dobson-Brewer frost-point hygrometer.

2. Aircraft measurements

A high-resolution spectrometer equipped with an automatic sun-follower has been installed in a Canberra aircraft. The design and operation of the instrument has been described by Houghton, Moss and Chamberlain (1958). Two changes have been made during the progress of the work on the 2·7 μ water-vapour band. Firstly, since considerable difficulty was experienced with the pen recorder during airborne observations, the arrangement for recording the signal on photographic film was improved so that the film records could be used in accurate analysis of the absorption. The film recorder responds faster than the pen recorder and so it has been possible to scan the spectrum more quickly. On nearly all the ascents described in this paper, the scanning speed has been 8 cm^{-1} sec^{-1}; the records shown in Fig. 1 have thus been scanned in about 8 sec. Secondly, the inside of the spectrometer could not be maintained sufficiently dry for there to be negligible absorption by the water-vapour it contained, even though a flow of dry nitrogen, obtained by boiling liquid nitrogen in a large Dewar vessel, was blown through the instrument.

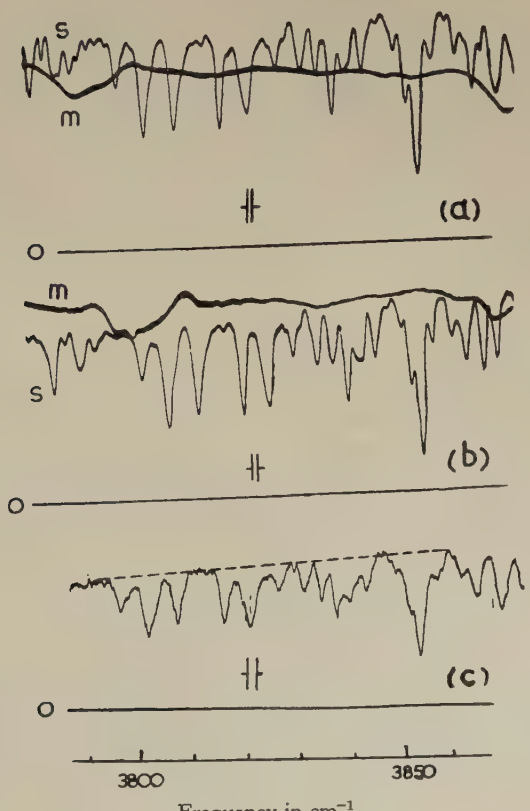


Figure 1. Examples of spectra showing m-monitor signal; s-signal from detector; o-zero of detector. Slit widths are indicated on each of the spectra. (a) and (b), solar spectra from aircraft on 12 June 1958 at approx. 1610 GMT, (a) from 45,000 ft, (b) 40,000 ft; (c) Spectrum recorded in altitude chamber: total pressure 122 mb, path-length of ppt. water-vapour approx. 5·5 μ.

To overcome this, two cylinders of compressed air, containing 600 ft^3 at 3,600 lb $in.^{-2}$, were installed in the bomb-bay of the aircraft and arranged to blow a much larger quantity of dry air through the spectrometer. Dry air from similar cylinders on the ground has been passed through the spectrometer for about an hour immediately prior to airborne measurements. To help further in keeping the instrument dry, surfaces inside the spectrometer have been coated with a silicone preparation to reduce adsorption of water and the covers have been made as leak-proof as possible, so that the dry air is forced to pass through the whole of the instrument, leaving only by the entrance slit.

The dryness of the spectrometer has been checked by mounting a source of light, a small 24 v bulb, directly on the entrance slit of the spectrometer, so that the absorption due to the water vapour present in the instrument itself alone is recorded. Records of this absorption have been obtained on the ground and on flights to high altitude. Although the absorption lines could still be seen on the records, the absorption is much smaller than that due to the atmosphere at the highest altitude reached. A correction has been made for this absorption and is described in Section 5 below.

Specimens of the records of absorption spectra obtained at different heights for the spectral region 3,790-3,860 cm^{-1} are shown in Fig. 1. For comparison a spectrum obtained in the laboratory has been included. Also shown on the records is the signal from a selenium photocell receiving the light reflected from the front surface of the prism in the spectrometer. This provides a monitor of the total light entering the spectrometer, and enables the background curve, the line corresponding to 100 per cent transmission, to be drawn in. When this has been done it is clear that there are a number of wavelengths at which the absorption for small amounts of water vapour is very small. The spectrum has been divided up into four groups of lines separated by these wavelengths. The groups, which are labelled A, B, C and D in Fig. 2 are thus effectively isolated from each other. The areas included between the absorption lines in these groups and the background curve have been measured. From these areas the path-length of water vapour to which each record corresponds has been deduced in the way described in the following sections.

During the ascents when spectra have been recorded, observations have also been made whenever possible of the air temperature using a standard flat-plate thermometer, and of the frost point using a Dobson-Brewer hygrometer.

3. Calculated absorption

The positions and relative intensities of the absorption lines due to water vapour in the region of 2·7 μ have been calculated by Benedict and Plyler (1951). Using their data, a spectrum has been constructed for the region 3,790-3,863 cm^{-1} (this is shown in Fig. 2). The lines have been drawn with a Lorentz shape and the relative intensities

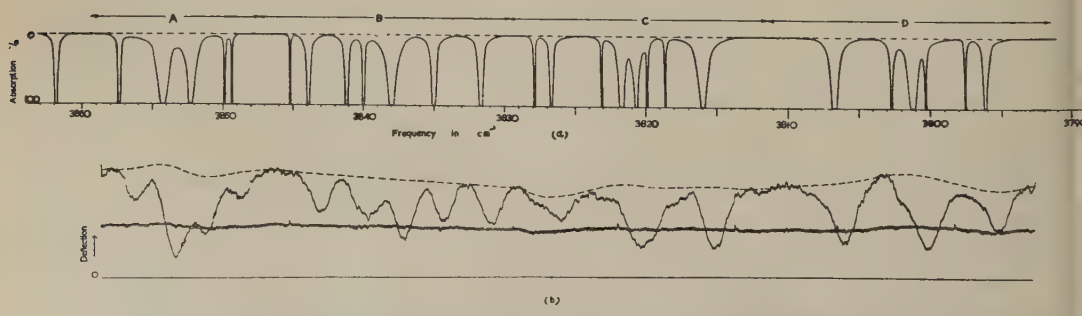


Figure 2. Absorption due to water vapour in a part of the 2·7 μ band. (a) calculated from data of Benedict and Plyler (1951); (b) observed in the solar spectrum from 35,000 ft.

have been multiplied by a factor such that the total absorption corresponds to that observed in the solar spectrum at 35,000 ft, which is also shown in Fig. 2. The shapes of the lines were calculated using a half-width of 0·006 cm^{-1}. Benedict (1956) gives a value of 0·06 cm^{-1} for the mean half-width of water-vapour lines at s.t.p. A half-width of 0·006 cm^{-1} thus corresponds to a pressure of 100 mb. However, it will be obvious from Fig. 2, that all the lines involved are ' black ' at their centres, and thus their profile would not be altered significantly if a slightly different half-width had been used; only the numerical factor by which the relative intensities have been multiplied would have been different. Causes of line broadening other than Lorentz are not important at the temperatures and pressures in the region of the atmosphere under consideration. In the gaps between the four groups of lines mentioned in Section 2, it will be seen that, for small path-lengths of water vapour, the transmission is greater than 99 per cent. For different path-lengths the integrated absorption for each of these groups has been calculated in the following way. The integrated absorption, or equivalent width W of a single spectral line of Lorentz shape, is given by the expression due to Ladenburg and Reiche (1911).

$$\int_{-\infty}^{\infty} A_\nu \, d\nu = W = 2\pi\gamma \, xe^{-x} \left[J_0(ix) - iJ_1(ix) \right] \quad . \quad . \quad (1)$$

where $x = \dfrac{Sw}{2\pi\gamma}$

and A_ν = fractional absorption at frequency ν

S = line strength

w = path-length of absorber

γ = half-width of the line

J_0 and J_1 = Bessel functions of zero and first orders respectively.

For large x ($x > 10$), the expression reduces to

$$W = 2\,(S\gamma w)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

According to simple kinetic theory, the half-width γ varies with pressure p and temperature T according to the expression

$$\gamma(p, T) = \gamma_0 \frac{p}{p_0} \left(\frac{T_0}{T} \right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (3)$$

where γ_0 is the half-width at p_0 and T_0. Using this expression Eq. (2) becomes

$$W = 2\,(S\gamma_0)^{\frac{1}{2}} \left(\frac{T_0}{T} \right)^{\frac{1}{4}} \left(\frac{pw}{p_0} \right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (4)$$

The strengths of the lines also vary with temperature owing to changes in the distribution of the molecules between the different energy levels. The population n_J of the energy level E_J from which a transition takes place, varies approximately according to the Boltzmann expression

$$n_J \propto \frac{1}{T^{3/2}} \exp\left(\frac{-E_J}{kT} \right)$$

where $T^{3/2}$ in the denominator comes from the rotational partition function (Herzberg 1945). Using the above equations the equivalent widths of the lines in each group have

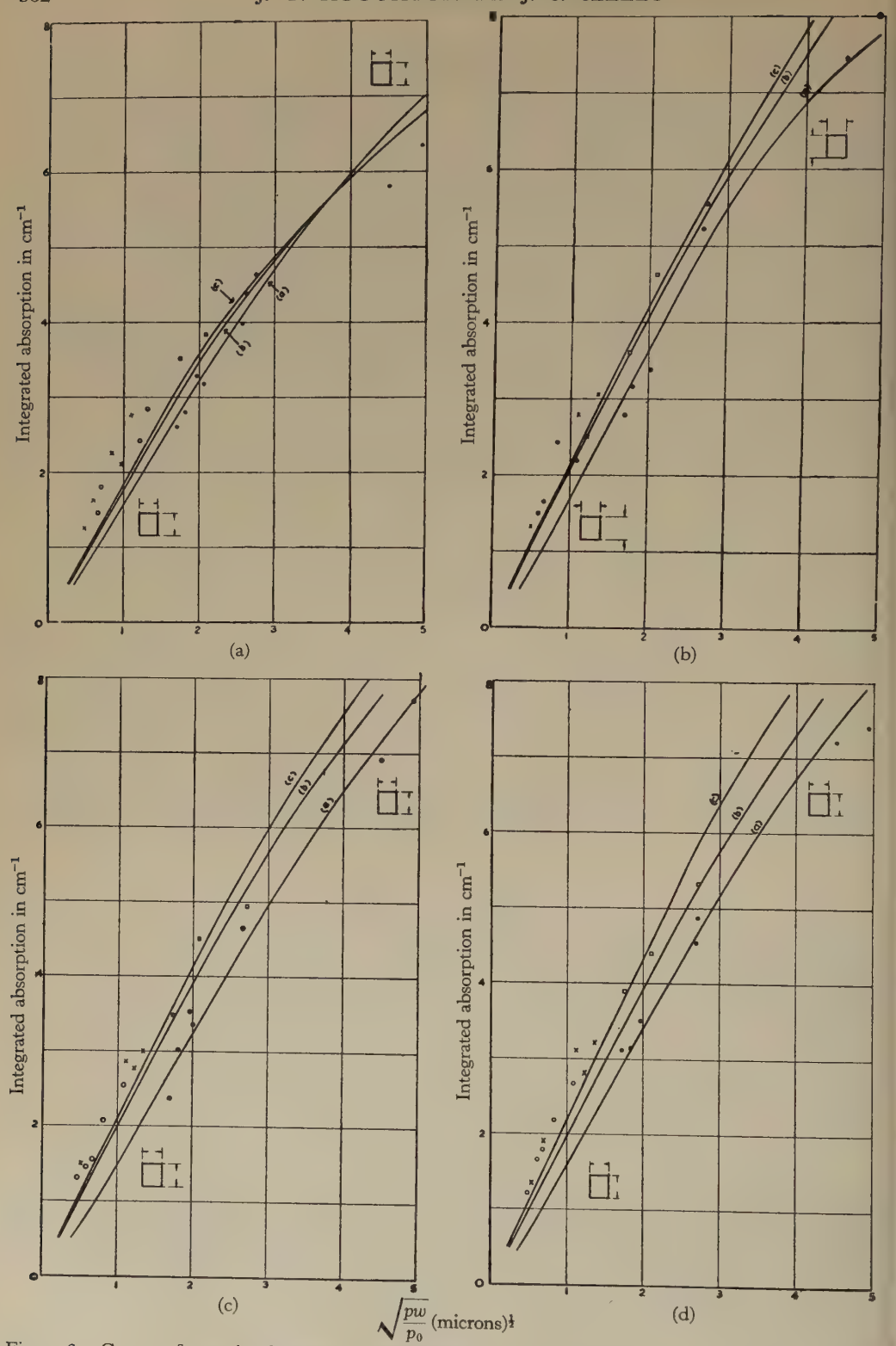


Figure 3. Curves of growth of groups of spectral lines, (a) Group A, (b) B, (c) C, (d) D. The lines represent calculations for (a) 1012 mb, 290°K; (b) < 300 mb, 267°K; (c) < 300 mb, 230°K. The points represent measurements under different conditions; = < 300 ○ mb, 290°K; × = 200-700 mb, 267°K; ⊡ = 300-800 mb, 290°K; ● = 1000 mb, 290°K; Experimental error is represented by the rectangles.

been calculated for different path-lengths, pressures and temperatures. The equivalent widths for each group have been summed and corrections made graphically for overlapping of the lines, to give values of integrated absorption. For large path-lengths of absorber, there is considerable absorption in the gaps between the groups owing to the wings of strong lines. The transmission between the groups has been calculated for different absorbing conditions. Since, when studying observed records of spectra, the background curve has always been drawn assuming that in these gaps there is perfect transmission, allowance has been made in the calculations for the amount lost by this assumption.

The results of the calculations are presented in the curves of Figs. 3 (a, b, c and d). Since at low pressure all the lines, and at higher pressure many of the lines, follow accurately the approximation of Eq. (4), the integrated absorption has been plotted against $\left(\frac{pw}{p_0}\right)^{\frac{1}{2}}$ with $p_0 = 1012$ mb. Were it not for overlapping of the lines, the graphs for low pressure would then be straight lines. Since the data of Benedict and Plyler (1951) only give relative strengths of the lines, each curve has been related to the actual quantities of water vapour in the absorbing path by comparing it with observations of absorption in the laboratory, as will be described in Section 4. The factors by which the line strengths as given by Benedict and Plyler have to be multiplied to give the best fit are not exactly the same for each group. They vary from 5·3 to 6·3 when w is measured in microns. This variation is most likely due to variations in the line-width from line to line. The assumption of constant line-width is certainly not an accurate one. Since, however, the variation for water vapour lines has not been measured, and since there is little to go on in the way of theoretical prediction, this assumption is the best that can be made at the moment. For the present study, this factor is not of great importance since variations in line-width will have very little effect on the shape of the curves of growth, and observational data have been used to determine the relative positions of the curves.

4. Laboratory measurements

Experimental data have been obtained on the rate of growth of the absorption groups A, B, C and D by two methods. The easiest was to use short columns of unenclosed laboratory air for absorption samples, the spectrometer itself being flushed with dry air from a high-pressure cylinder. The absorption due to the residual water-vapour content of the spectrometer was measured by placing a source at the entrance slit. Values of temperature, pressure and relative humidity were recorded so that the amount of water vapour in the column of laboratory air could be calculated. The shortest path-length that could be conveniently used was 70 cm; under normal conditions of humidity the absorption produced by this column was greater than that observed at high altitude in the solar spectrum. To provide data on the absorption produced by small amounts of water at low pressure, extensive measurements of rates of growth have been made in an altitude chamber. The chamber used was sufficiently large to allow the spectrometer to be placed inside. The optical path of the spectrometer (2·8 m) was fully exposed to the conditions inside the chamber to form part of the overall absorption path of 8 m. The chamber could be supplied with very dry air at two temperatures, ambient or cooled to 35°C below ambient; in each case the pressure could be maintained at any suitable value within the range 1000 mb to less than 100 mb. Fig. 4 shows the general arrangement inside the chamber and the positions at which temperature, frost point and pressure were measured. Preliminary tests showed that large inconsistencies in absorption values were recorded after a pressure change had been made, unless conditions were allowed to stabilize at the new pressure for about 20 minutes. This showed clearly that the air in the absorption column could be subject to non-uniform mixing without this being obvious from the indications of temperature and frost point.

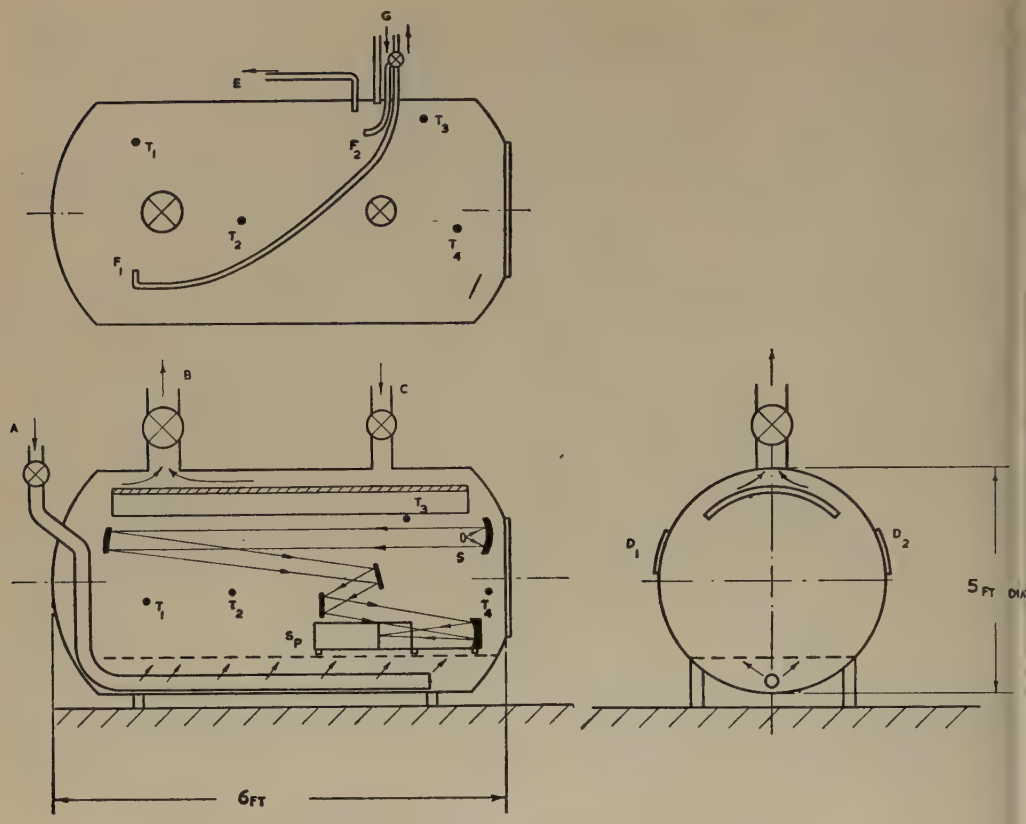


Figure 4. Disposition of equipment in altitude chamber.

A = dry air main supply
B = suction main
C = atmospheric bleed (if required)
D_1, D_2 = bulkheads for electrical and pipe connexions
E = to manometer
F_1, F_2 = frost point measured here
G = pipes to frost-point hygrometer (pumped)
S = source
S_p = spectrometer
T_1, T_2, T_3, T_4 = temperatures measured here.

A typical record obtained in the pressure chamber is compared in Fig. 1 with sections of the solar spectrum observed at different altitudes showing similar amounts of absorption. The absorption of each group is evaluated as $\frac{\int y d\nu}{\bar{y}}$, where $\int y d\nu$ is the area under the background curve and $\bar{y}$ is the mean level of the background curve. To make comparison with theoretical curves, the results of the experiments made in the chamber have been separated into the following categories :

(i) Low pressure ($<$ 300 mb), ambient temperature 290°K.
(ii) Medium pressure (200-700 mb), 267°K.
(iii) Medium pressure (300-800 mb), 290°K.
(iv) High pressure ($\simeq$ 1012 mb), 290°K.

The measured values of absorption are plotted in Fig. 3, where they can be compared with theoretical curves of growth constructed, as described in Section 3, for the following conditions :

(a) Pressure 1,012 mb, temperature 290°K.
(b) Low pressure (< 300 mb), 267°K.
(c) Low pressure (< 300 mb), 230°K.

Errors in experimental values of absorption are assessed as being due to uncertainties in the construction of the background curve and in the measurement of areas under this curve. The combined error is estimated as not more than $\pm 0{\cdot}15$ cm^{-1} within the range 1·0 to 7·0 cm^{-1}. In a few cases these limits may have been exceeded when the electrical signal-to-noise ratio was unusually low or when, for spectra recorded in the aircraft, the background level was subject to large fluctuations. The effects of experimental errors in the measurement of frost points or relative humidities may be assessed as corresponding to $\pm$ 10 per cent in $\left(\frac{pw}{p_0}\right)^{\frac{1}{2}}$ when this has a value near 1·0, and $\pm$ 3 per cent when it is about 4·0.

In the construction of Fig. 3 the theoretical curves of growth have been matched to the experimental data at high pressure. It will be seen that the experimental points at low pressure and small path-lengths consistently show more absorption than expected. The difference between the experimental points and the theoretical curves is at the limit of the possible error of measurement mentioned above. It cannot be associated with either of the experimental methods since the high-pressure data were obtained in both the laboratory and the altitude chamber, and the low-pressure data obtained at the two temperatures were reproduced satisfactorily on several occasions. There is doubt about the complete uniformity of mixing for both methods, but this cannot easily be estimated. Because of this discrepancy, there is some uncertainty in the absorption produced by very small amounts of water vapour. This will be taken into account in assigning quantities to the absorptions measured in the stratosphere.

5. Results

The quantity of water vapour in the optical path of the spectrometer has been measured at various altitudes. Mean values taken from the results for all groups are shown in Table 1. The water-vapour content of the spectrometer was less during pre-flight conditions on the ground because a better dry-air supply could be maintained . The difference in dryness between winter and summer is expected since under normal conditions of humidity and temperature an equivalent air path at ground level would contain nearly 19 ppt μ in summer and 10 μ in winter. It is possible that the spectrometer was not always so dry as the figures in Table 1 indicate, and it can be assumed that quantities up to twice these amounts could have been present on occasions.

TABLE 1. Path-length of water vapour in optical path of spectrometer, 2·8 m long

Altitude ft	Pressure mb	Average $\left(\frac{pw}{p_0}\right)$ from A B C D (microns)		w in microns ppt H_2O vapour at pressure p	
		Winter	Summer	Winter	Summer
Ground level	1012	0·13	0·20	0·13	0·20
30,000	301	0·08	0·12	0·27	0·41
40,000	187	0·05	0·10	0·26	0·53
45,000	147	0·04	0·09	0·30	0·60

From the curves of Fig. 3, the measured values of integrated absorption from spect obtained at different altitudes have been used to read off values of $\frac{pw}{p_0}$. At the lo pressures involved, the integrated absorption is quite accurately a function of this quantit The value of $\frac{pw}{p_0}$ at the nth altitude (counting the highest altitude as the first and th top of the atmosphere as the zeroth) may be written

$$\left(\frac{pw}{p_0}\right)_n = \left(\frac{pw}{p_0}\right)_s + \sum_{1\ldots n} \frac{(\bar{p}(n, n-1)\, w(n, n-1))}{p_0} \operatorname{cosec} \theta \quad . \quad . \quad (4)$$

where $\left(\frac{pw}{p_0}\right)_s$ = appropriate term shown in Table 1 related to the quantity of wat vapour in the optical path of the spectrometer.

θ = solar elevation.

$w(n, n-1)$ = water-vapour path-length in a vertical column between the tw levels at which the pressures are p_n and p_{n-1} respectively.

$\bar{p}(n, n-1)$ = mean pressure between these two levels.

Between adjacent levels the mixing ratio has been assumed to have a constant valu $r(n, n-1)$ so that w and $\bar{p}$ can be written

$$w(n, n-1) = \frac{1}{g} r(n, n-1)(p_n - p_{n-1}) \quad . \quad . \quad . \quad (5)$$

$$\bar{p}(n, n-1) = \tfrac{1}{2}(p_n + p_{n-1}) \quad . \quad . \quad . \quad . \quad (6)$$

Eq. (5) may now be written

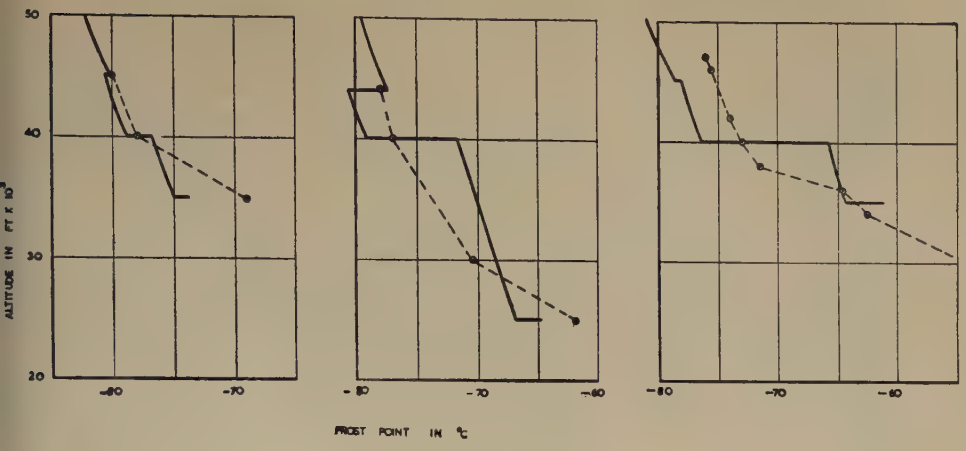
$$\left(\frac{pw}{p_0}\right)_n = \left(\frac{pw}{p_0}\right)_s + \sum_{1\ldots n} \frac{(p_n^2 - p_{n-1}^2)}{2\, p_0\, g} r(n, n-1) \operatorname{cosec} \theta \quad . \quad . \quad (7)$$

for the nth level, and

$$\left(\frac{pw}{p_0}\right)_1 = \left(\frac{pw}{p_0}\right)_s + \frac{p_1^2}{2\, p_0 g} r(1, 0) \operatorname{cosec} \theta$$

for the highest level, where it is assumed that the mixing ratio $r(1, 0)$ is constant to th top of the atmosphere.

The results are shown in Table 2. The limits to the values of integrated absorptio shown in column 6 have been assigned both by inspecting the quality of the record an the accuracy with which the actual area could be measured, and by comparing record obtained successively at the same altitude on the same ascent. All the areas given are th mean of from two to four records except for those at 35,000 ft and 40,000 ft on 10 Decembe 1957 when six records were available, and for those on 17 and 19 December 1957 whe only one record was available. The mean value of $\left(\frac{pw}{p_0}\right)_n - \left(\frac{pw}{p_0}\right)_s$ in column 7 is a average over the values deduced from the different groups weighted by considerati of the limits in column 6. The second limit in column 7 has been assigned by inspectio of the agreement between the values for the different groups. The first limit given, whic is negative, represents the extreme of the uncertainty in the water-vapour path-length represented by small absorptions.



(a) 10 December 1957 (b) 21 January 1958 (c) 12 June 1958

Figure 5. Comparison with frost-point measurements.

⊙ = measured with frost-point hygrometer
— = calculated from spectroscopic data assuming constant mixing-ratio between levels.

(For flight labelled 12 June 1958, the frost points were measured on the previous day (11 June) at the same time of day).

A comparison can be made between the mixing ratios calculated from the spectroscopic measurements and measured values of frost points for the flights made on 10 December 1957, 21 January and 12 June 1958. The appropriate values of mixing ratio given in Table 2 have been converted into calculated equivalent frost points and these are compared with measured values in Fig. 5, which shows that reasonable agreement has been reached. The possible errors have been estimated as $\pm$ 3°C in measured frost points, and $\pm$ 2°C in the frost points calculated from the spectroscopic data.

6. Discussion

It will be seen from Table 2 that measurements of the water-vapour mixing ratio in the stratosphere show a fairly constant value of about 3×10^{-6}. The variations found between different occasions over England are hardly outside the possible error and no significance can be attached to them. The one set of observations at 45,000 ft at a latitude of 29°N appears to show more absorption, which might be expected, as these observations were made well below the tropopause which lay at about 100 mb. The record obtained on this occasion was a poor one, however, as is indicated by the possible error attached to the mixing ratio, and little significance can be attached to the difference between this measurement and those over England.

A mixing ratio of 3×10^{-6} corresponds to a frost point at 45,000 ft of — 78°C. This is 3-5°C higher than that normally measured at this height by the frost-point hygrometer (see Murgatroyd, Goldsmith and Hollings (1955)). This disagreement is partially removed if the absorption values obtained for small path-lengths in the altitude chamber, and which lie consistently above the curves in Fig. 3, are used to compute the results. Since the altitude-chamber observations are also dependent on measurements with the frost-point hygrometer, a possible explanation of this would be a systematic error in the frost-point measurements of about 2°C in both cases. However, it would seem unlikely that any error would be so large at about — 35°C, the values used in the altitude chamber, as at — 80°C. It will be seen from Fig. 5 that for larger path-lengths agreement between the two methods is as good as can be expected considering the possible errors of measurement and the variability of the atmosphere.

TABLE 2. MEASUREMENTS OF WATER-VAPOUR MIXING-RATIO IN THE ATMOSPHERE

Date Approx. time and place	Altitude ft	Pressure mb	Solar elevation θ	Absorption group	Integrated absorption cm^{-1}	Mean value of $\left(\frac{pw}{p_0}\right)_n - \left(\frac{pw}{p_0}\right)_s$ (See Eq. 5) microns ppt. H_2O vapour	Diff. between levels $= \frac{\bar{p}(n, n-1)\, w(n, n-1)}{p_0}$ microns ppt. H_2O vapour	Mean mixing-ratio $r(n, n-1)$ from Eq. (8) $\times 10^6$
10 December 1957 1130 GMT over southern England	35,000	238	15·3°	A	2·9 ± 0·1			
				B	3·6 ± 0·25			
				C	3·7 ± 0·15	2·80 − 0·40 ± 0·30		
				D	4·05 ± 0·15			
							0·33 − 0·04 ± 0·11	3·0 − 0·4 ± 1·0
	40,000	187	15·7°	A	2·25 ± 0·1			
				B	2·8 ± 0·25			
				C	2·65 ± 0·15	1·50 − 0·25 ± 0·10		
				D	2·8 ± 0·2			
							0·18 − 0·04 ± 0·07	2·7 − 0·6 ± 1·0
	45,000	147	16°	A	1·6 ± 0·1			
				B	1·6 ± 0·3			
				C	1·95 ± 0·1	0·85 − 0·10 ± 0·15	0·26 − 0·03 ± 0·05	2·5 − 0·3 ± 0·5
				D	2·4 ± 0·1			
17 December 1957 1200 GMT over southern England	40,000	187	19°	A	2·1 ± 0·2			
				B	3·0 ±0·4			
				C	2·9 ± 0·25	1·70 − 0·25 ± 0·30	0·55 − 0·08 ± 0·10	3·1 − 0·5 ± 0·6
				D	3·4 ± 0·25			
19 December 1957 1220 GMT over southern England	44,000	155	16·6°	A	2·1 ± 0·2			
				B	2·1 ± 0·4			
				C	2·4 ± 0·2	1·20 − 0·15 ± 0·15	0·34 − 0·05 ± 0·05	2·9 − 0·4 ± 0·4
				D	2·3 ± 0·3			
20 January 1958 1240 GMT over southern England	48,000	127	18·5°	A	1·8 ± 0·1			
				B	1·9 ± 0·1			
				C	2·1 ± 0·1	0·95 − 0·10 ± 0·15	0·30 − 0·03 ± 0·04	3·8 − 0·4 ± 0·6
				D	2·45 ± 0·1			

TABLE 2 (continued)

Date Approx. time and place	Altitude ft	Pressure mb	Solar elevation θ	Absorption group	Integrated absorption cm^{-1}	Mean value of $\left(\frac{pw}{p_0}\right)_n - \left(\frac{pw}{p_0}\right)_s$ (See Eq. 5) microns ppt. H_2O vapour	Diff. between levels $= \frac{\bar{p}(n, n-1)\, w(n, n-1)}{p_0}$ microns ppt. H_2O vapour	Mean mixing-ratio $r(n, n-1)$ from Eq. (8) $\times 10^6$
21 January 1958 1140 GMT over southern England	25,000	376	17·6°	A	5·4 ± 0·5			
				B	7·0 ± 1·0			
				C	6·7 ± 0·5	12·9 ± 1·0		
				D	6·3 ± 0·5			
							3·35 + 0·08 ± 0·33	6·3 + 0·2 ± 0·6
	40,000	187	18·25°	A	2·5 ± 0·2			
				B	2·9 ± 0·4	1·75 − 0·25 ± 0·10		
				C	2·8 ± 0·2			
				D	2·9 ± 0·2			
							0·10 − 0·02 ± 0·08	1·8 − 0·4 ± 1·4
	44,000	155	18·75°	A	2·2 ± 0·2			
				B	2·4 ± 0·4			
				C	2·4 ± 0·2	1·40 − 0·20 ± 0·15	0·45 − 0·04 ± 0·04	3·7 − 0·4 ± 0·4
				D	2·8 ± 0·2			
12 June 1958 1610 GMT over southern England	35,000	238	41·1°	A	3·3 ± 0·1			
				B	4·0 ± 0·1			
				C	3·6 ± 0·4	3·20 − 0·40 ± 0·30		
				D	4·4 ± 0·2			
							1·65 − 0·20 ± 0·30	15·1 − 1·9 ± 2·8
	40,000	187	38·1°	A	2·05 ± 0·05			
				B	1·9 ± 0·1			
				C	2·05 ± 0·05	0·90 − 0·10 ± 0·15		
				D	2·3 ± 0·05			
							0·22 ± 0·15	3·3 ± 2·0
	45,000	147	35°	A	1·6 ± 0·1			
				B	1·65 ± 0·15			
				C	1·65 ± 0·1	0·58 − 0·10 ± 0·10	0·33 − 0·06 ± 0·06	3·1 − 0·6 ± 0·6
				D	1·95 ± 0·15			
29 November 1957 1000 GMT Approx. 29°N (flight based on Malta)	45,000	147	34°	A	2·15 ± 0·3			
				D	2·15 ± 0·3	1·00 − 0·15 ± 0·30	0·56 − 0·08 ± 0·17	5·2 − 0·7 ± 1·5

Acknowledgments

Our thanks are due to the pilots, crew and ground staff of Radio Flight, Royal Aircraft Establishment, for their willing co-operation with the airborne measurements. The observations with the frost-point hygrometer were made by Mr. M. Kerley and Mr. N. C. Helliwell of the Meteorological Research Flight, Meteorological Office. Mr. T. D. F. Hawkins acted as airborne observer on some of the flights, and Mr. I. D. Birch took part in the laboratory measurements. The observations over North Africa were made while the aircraft was based at Luqa Airport, Malta, where members of the Transit Flight gave every assistance. We are also indebted to Mechanical Engineering Department, Royal Aircraft Establishment, who allowed access to the altitude chamber and associated facilities and provided generous assistance during its use.

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Atmospheric extinction of infra-red radiation*

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(Manuscript received 29 January 1960)

Summary

The effects of scattering on the transmission of 2·6 to 12 micron radiation are analysed as a continuation of an earlier paper. Monochromatic extinction coefficients are determined on the basis of the original haze models and of the exact values of the complex index of refraction for water. The individual extinction cross-sections are computed with a high degree of accuracy by means of a modified van de Hulst expression.

The results are in good agreement with existing experimental determinations of window transmission except in the 8 to 12 μ region, where they support Roach and Goody's (1958) conclusion on the importance of wing absorption by molecular lines. The computed continuous spectrum of haze extinction presented graphically and in tables, further reveals interesting details which will be useful in determining the relative importance of absorption and scattering in atmospheric infra-red radiation.

A simple cloud model is also considered, and it is shown that even tenuous clouds may be quite opaque to infra-red radiation over a wide band. In the window regions, clouds and dense hazes scatter a considerable portion of the incident radiation, thereby contributing to the infra-red sky background usually attributed to black-body emission.

1. Introduction

In a previous paper (Deirmendjian 1959b) the role of haze in the attenuation of infra-red radiation by scattering was examined, assuming that natural haze consists of water droplets of given size distributions and concentrations. The continuous spectrum of attenuation was then computed for wavelengths up to 2·25 μ where the index of refraction of water may be approximated by a real and constant number.

For infra-red radiation of a longer wavelength the index is complex and it varies with the wavelength. This means that water droplets both scatter and absorb the incident radiation in varying amounts depending on the wavelength. Since the combined extinction cross-section depends critically on the complex refractive index as well as the relative size, it is important to compute this quantity separately for each wavelength and integrate it with respect to the assumed size-distribution function. The cross-section can be obtained from rigorous electromagnetic theory by using the well-known expressions originally derived by G. Mie. These expressions contain series involving Bessel functions of complex argument, in such a form that an integration with respect to size cannot be easily carried out. The Mie solution has been used by Johnson and Terrel (1955) to obtain the extinction cross-section for selected wavelengths or sizes by fitting step functions to the real and imaginary parts of the refractive index.

An alternative method is based on van de Hulst's approximation for transparent spheres (van de Hulst 1957), in which the absolute value of the complex index is assumed to approach unity. As will be shown below, the corresponding expressions may be used with good accuracy provided that they are corrected by means of an appropriate factor to take account of the true refractive index. The modified expressions may then be used to interpolate the extinction with respect to size and to index, and to integrate this extinction over a size distribution, either graphically or analytically.

* Presented in summary at the Radiation Symposium of the International Association of Meteorology and Atmospheric Physics, Oxford, July 1959.

2. Correction factors in the approximate theory

With minor changes in notation van de Hulst's (1957, p. 179) expression for the total extinction cross-section for partially absorbing spheres may be written in the form :

$$K_\lambda(z, m) = 2 - 4\frac{\cos b}{z} e^{-x} \sin(z - b) + 4\left(\frac{\cos b}{z}\right)^2 [\cos 2b - e^{-x} \cos(z - 2b)], \quad (1)$$
$$|m - 1| \to 0$$

in which

$m \equiv n - in'$, is the complex index of refraction

$z \equiv \frac{4\pi a}{\lambda}(n - 1) = 2\alpha(n - 1)$, is van de Hulst's normalized size parameter, where

λ is the wavelength and a the radius of the sphere,

$b \equiv \arctan \frac{n'}{n - 1}$, is the absorption parameter,

$x = z \tan b = 2a\gamma_w$, is the absorption within the sphere along the axial ray, γ_w being the absorption coefficient for water.

$\pi a^2 K(z, m)$, is the real extinction cross-section (absorption plus scattering) of the sphere for monochromatic radiation.

It can be seen by inspection that (1) reduces to the equivalent expression (11) for non-absorbing spheres when $n' = 0$, and to the geometric optics value of 2 when $n' \to \infty$ (opaque sphere). For small values of n' and $n - 1$ van de Hulst (1957) has shown that (1) reproduces the correct general behaviour of the curve $K(z, m)$ as a function of z. However, the magnitude of the extinction falls short of the exact Mie values when the correct refractive index of water is used.

It is shown elsewhere (Deirmendjian 1957, 1959a, 1959b) that a van de Hulst-type approximation may be extended to the case of non-absorbing spheres with $m = 1{\cdot}33$, by means of an appropriate correction factor. For partially absorbing spheres, a comparison of the extinction cross-sections computed by Johnson and Terrel (1955) with those obtained from expression (1), indicated that a similar correction factor may exist in this case also. Such a factor must depend on both the real and imaginary parts of the index, and on the relative size, and it should apply over a wide range of z. Actually, it was found that no single factor can be applied in the entire range of z. After many trials the following four expressions were found to give the best results in the ranges indicated :

$$D_1(b, z) = \frac{n - 1}{2n} \frac{5(n - 1)}{4{\cdot}08} [f(b) + 1] - \frac{5(n - 1) - z}{5(n - 1) f(b)}; \quad z \leqq 5(n - 1) < \frac{4{\cdot}08}{1 + 3\tan b} \quad (2)$$

$$D_2(b, z) = \frac{n - 1}{2n} [f(b) + 1] \frac{z}{4{\cdot}08}; \quad 5(n - 1) \leqq z \leqq \frac{4{\cdot}08}{1 + 3\tan b} \quad (3)$$

$$D_3(b) = \frac{n - 1}{2n} \frac{f(b) + 1}{1 + 3\tan b}; \quad \frac{4{\cdot}08}{1 + 3\tan b} \leqq z \leqq \frac{4{\cdot}08}{1 + \tan b} \quad (4)$$

$$D_4(b, z) = \frac{n - 1}{2n} \frac{f(b) + 1}{f(b)} \frac{4{\cdot}08}{z}; \quad z > \frac{4{\cdot}08}{1 + \tan b} \quad (5)$$

where, $$f(b) \equiv (1 + \tan b)(1 + 3\tan b) = 1 + 4\frac{n'}{n - 1} + 3\left(\frac{n'}{n - 1}\right)^2 \quad (6)$$

and the functions D_i given by (2) to (5) are correction factors such that

$$K'(z, m) = (1 + D_i) K(z, m), \quad (7)$$

where $K(z, m)$ as in (1), represents the corrected total extinction cross-section in the range, and may be used as an interpolation formula.

The above-mentioned corrections have not been derived on the basis of physical theory. The only justification in their use is that they give very accurate values (within ± 4 per cent) of the exact, Mie series, values, for various choices of z, n and n', in the range $1 < n \leqq 1{\cdot}50$ and $0 \leqq n' \leqq 0{\cdot}25$. However, the modified cross-section (7) is a continuous function of z through the limits of the various ranges, and the expressions D_i vanish with $n - 1$, and as $z \to \infty$. Further, for non-absorbing spheres ($n' = 0$) the formulae reduce to :

$$D_1(n, z) = \frac{n-1}{n}\frac{5(n-1)}{4{\cdot}08} - 1 + \frac{z}{5(n-1)}; \quad z \leqq 5(n-1) < 4{\cdot}08 . \qquad (8)$$

$$D_2(n, z) = \frac{n-1}{n}\frac{z}{4{\cdot}08}; \quad 5(n-1) \leqq z \leqq 4{\cdot}08 . \qquad (9)$$

$$D_4(n, z) = \frac{n-1}{n}\frac{4{\cdot}08}{z}; \quad z > 4{\cdot}08 \qquad (10)$$

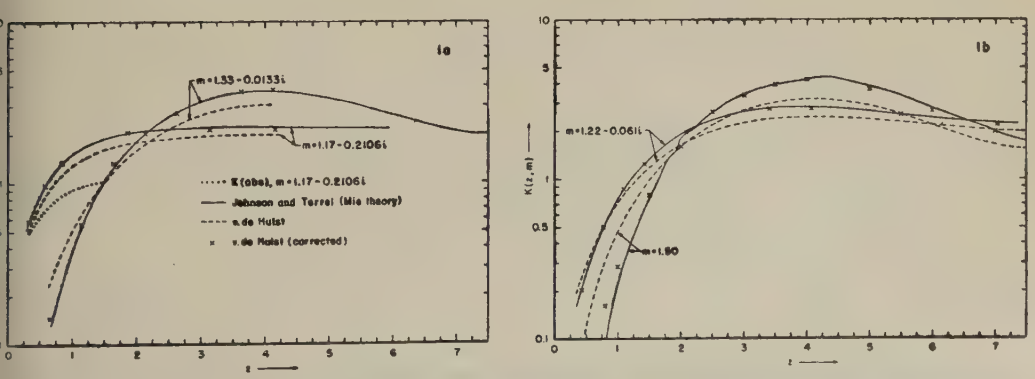
The number 4·08 represents the critical relative size at which dielectric spheres show the first and most important resonance peak in scattering cross-section (Penndorf 1956).

When (8), (9) and (10) are substituted in (7), together with the van de Hulst cross-section for non-absorbing spheres, given by

$$K(z) = 2 - \frac{4\sin z}{z} + 4\frac{(1-\cos z)}{z^2}, \qquad (11)$$

the corrected values turn out to check equally well with the corresponding exact values.

The degree of accuracy of these interpolation formulae is illustrated by the examples in Fig. 1 (a) and (b). The full curves represent the smoothed, exact extinction cross-section tabulated by Johnson, Eldridge and Terrel (1954) and Penndorf (1956, Table A.9), for various values of m. The dash curves are obtained by means of the original van de Hulst expressions (1) and (11), and the corrected values at certain points are shown by crosses. It will be noted that the corrections reproduce the exact theory rather well. For large values of n and small z, we have $(1 + D_1) < 1$, that is the van de Hulst approximation overestimates the cross-section, and this is best illustrated in the curves for $m = 1{\cdot}50$. At the first resonance peak at $z = 4{\cdot}08$ the cross-section for non-absorbing spheres given by (11) must always be multiplied by the factor $1 + (n-1)/n$ for any value $1 < n \leqq 1{\cdot}5$, to get the exact magnitude. Similar remarks apply to absorbing spheres.



Figures 1 (a) and (b). Comparison of extinction cross-sections determined by the exact and approximate theories.

There must be some physical reasons for these features which will undoubtedly become clear when the van de Hulst theory is extended to take into account the refraction and reflection of rays at the interface of the sphere and the medium. Such work is in progress although the results are not yet available (Sekera 1959). However, the fact that different corrections must be used, depending on the range of z, shows that such effects are complicated : they vary in relative importance according to the size of the sphere, and the physical characteristics of the medium.

3. Evaluation of the extinction coefficient

By means of the interpolation formulae, described above, the extinction cross-section may be estimated with considerable accuracy and little effort, provided one uses the exact values of n and n'. When a wide spectral range is involved, this method is preferable to the Mie series, where, for economy of computation, one has to use a constant index over a range of wavelengths, (cf. Johnson and Terrel 1955). In this case the advantage of using the cumbersome exact theory is lost since even small variations in the index reflect in the computed cross-section. For the purposes of the present work we chose the following wavelengths and indices of refraction :

λ	2·6 μ	3·0 μ	3·5 μ	3·9 μ	5·0 μ	6·04 μ
m	1·252	1·446–0·068 i	1·423–0·0128 i	1·353	1·331–0·0169 i	1·312–0·1372 i

λ	8·6 μ	10 μ	12 μ	14 μ
m	1·282–0·0476 i	1·212–0·0601 i	1·165–0·2058 i	1·309–0·3582 i

Except for λ 2·6 μ and 3·9 μ, where we have neglected the small imaginary part, these values are exactly as reported by Centeno (1941).

The extinction cross-sections were computed at each wavelength according to formula (7) with the help of a desk computer and tables, at sufficient intervals in the range of interest to obtain size-variation curves like those of Fig. 1. The corresponding monochromatic extinction coefficient β_λ, per unit path length in a homogeneous haze layer, is obtained by integration with respect to the size distribution, that is

$$\beta_\lambda = \int n(a)\, \pi a^2 K'(a, m)\, da \quad . \quad . \quad . \quad (12)$$

where $n(a)$ is the size distribution, and $K'(a, m)$ is given by the equivalent value of $K'(z, m)$. The actual integration was carried out numerically using the distribution models and the method described earlier (Deirmendjian 1959a; 1959b). The haze models are also shown graphically on Fig. 2. Certain parts of the integration (12) can be performed analytically, but the resulting number of terms, some of which involve untabulated transcendentals, renders this method impractical. Straightforward numerical integration is less laborious and quite reliable since the integrand is a continuous function.

It should be noted that, depending on the wavelength, the integration (12) in this case mostly involves various portions *to the left* of the maximum in the curves illustrated in Fig. 1. Therefore any estimate of the total extinction on the basis of the asymptotic value $K \to 2$ would give entirely erroneous values.

Examples of the computed extinction coefficient per km of horizontal path are shown under Table 1 and on Fig. 3, indicated by the labels $A(a_2)$, $B'(a_2)$ and $C(a_2)$ where a_2 is the radius of the largest particle. The coefficients are such that $\exp[-\beta_\lambda L]$, where L is the horizontal path in km, gives the sea-level transmission at the wavelength indicated. Also, if the scale height of the haze atmosphere is close to 1 km the same coefficients will serve as the normal optical thickness of the haze atmosphere at sea level, from which the vertical or slant transmission can be determined in the usual way.

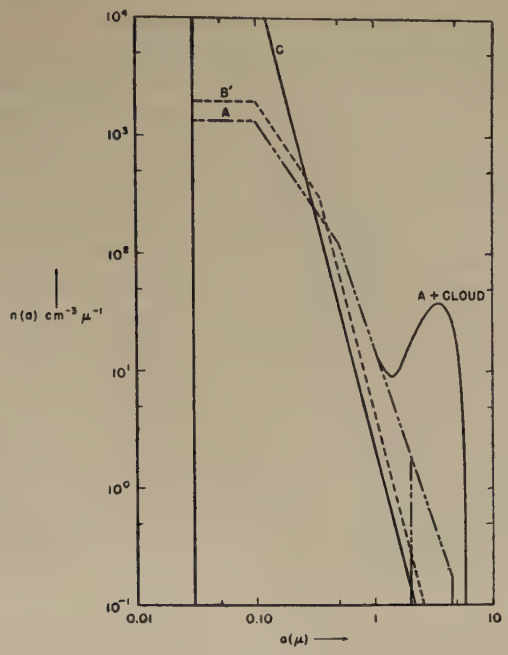


Figure 2. Droplet size-distribution models used in computing the infra-red extinction.

TABLE 1. SEA-LEVEL MONOCHROMATIC EXTINCTION COEFFICIENTS β_λ km^{-1}

Model/λ	2·6	3·0	3·5	5·0	6·04	8·6	10·0	12·0	14·0 μ
				Haze Models					
A (2 μ)	0·0557	0·1228	0·0849	0·0268	0·0464	0·0108	0·0086	0·0399	0·0362
B′ (2 μ)	0·0140	0·0389	0·0207	0·0068	0·0198	0·0033	0·0030	0·0093	
C (2 μ)	0·0072	0·0222	0·0111	0·0037	0·0101	0·0025	0·0021	0·0071	0·0109
A (4·48 μ)	0·1835	0·2259	0·2127	0·1234	0·1646	0·0766	0·0298	0·0736	0·0879
B′ (4·48 μ)	0·0269	0·0499	0·0342	0·0160	0·0295	0·0083	0·0050	0·0126	
C (4·48 μ)	0·0137	0·0277	0·0179	0·0083	0·0150	0·0046	0·0030	0·0088	0·0134
				Thin-Cloud Model					
	13·0	9·77	11·1	13·1	9·69	5·81	3·40	4·54	6·82
				Model A (4·48 μ) + $\frac{1}{10}$ Cloud					
	1·48	1·20	1·32	1·43	1·13	0·657	0·369	0·528	

The numbers shown under ' haze models ' in Table 1 are directly comparable with, and present an extension of, the previous estimate (Deirmendjian 1959b, Table 1). It will be noted that at λ 3·5 μ the present computation based on the true complex index of refraction instead of 1·33, results in much larger values of the extinction than the previous estimate. In other words, at this and longer wavelengths, one cannot estimate the haze effect correctly without considering the exact values of both the real and imaginary parts of the index.

The wavelength dependence of the extinction is illustrated on Fig. 3, which covers the entire spectrum from λ 0·60 μ to 12 μ. For comparison, the values of $n - 1$ and n' for liquid water are shown on the same scale by the dotted and dash curves, respectively. The full curves labelled according to the models (see Fig. 2) indicate the sea-level extinction per kilometer to be expected from hazes of this type, excluding the effect of molecular

absorption lines and bands. In a first approximation, gaseous absorption by molecules of water vapour and carbon dioxide may be assumed independent, and their effect may be superimposed linearly on the truly *continuous haze background extinction*. The amount of detail in the shape of the curves on Fig. 3 is limited by the computed points (indicated by open dots).

It is clear from Fig. 3 that the haze-extinction component in the real atmosphere is not a monotonic function of the wavelength in the region considered, and that it does not quite follow either the real or the imaginary part of the refractive index of water. There are band-like regions of maximum and window-like regions of minimum extinction as well as regions of steep gradients. In as much as natural haze may be represented by water or water-coated particles, this type of extinction must exist in varying intensities on all clear days. The curves further indicate the great sensitivity of haze extinction on the size distribution and the range of sizes in the particles. Note for example that for a model C distribution (Fig. 2) with about 2,300 particles per cm^3, the extinction is very tenuous compared to that of model A, with only 275 particles. Also increasing the radius of the largest particles from 2 to 4·48 μ in model A, which means adding about only 1 particle per cm^3 in this range, results in tripling the extinction at λ 10 μ. On the other hand, increasing the overall concentration of particles without changing the distribution or the range, simply means an increase in the extinction coefficients *at all wavelengths by the same factor.*

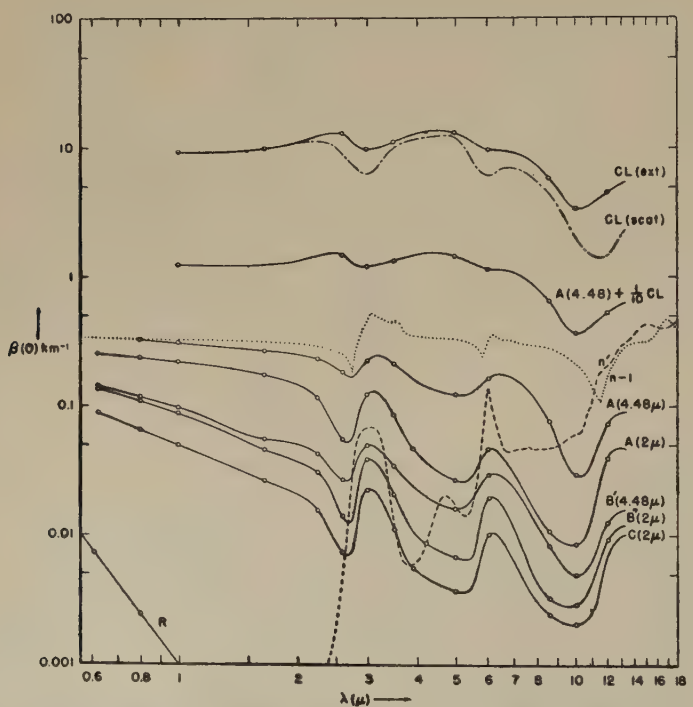


Figure 3. Computed spectrum of the sea-level horizontal extinction coefficient due to droplets only.

It should be emphasized that the extinction curves in Fig. 3 correspond to likely conditions near sea level consistent with observed values of extinction and aureole intensities in the visible spectrum (Deirmendjian 1959a, 1959b). At higher levels, the magnitudes of the horizontal extinction coefficients must decrease with the particle concentration. In particular, if the vertical distribution is independent of the size distribution, the reduction is given by a simple factor (op. cit.).

4. Comparison with observations and discussion

A comparison of the computed infra-red extinction with some values deduced from observation is made on Fig. 4, in which the coordinates are identical with those in Fig. 3. The values indicated by open dots are adapted from the paper by Gebbie and co-authors (1951), on the basis of their transmission curve per sea mile, *assuming an exponential monochromatic absorption*. Only the maximum transmission points in various windows were used. The crosses are taken directly from the work of Roach and Goody (1958) based on solar infra-red spectrobolometry by means of a Golay cell, and given in terms of the extinction per atmosphere in the 8-12 μ region. In so far as the scale height of the haze atmosphere is about 1 km, the two sets of observations are comparable. The numbers shown after the labels in Fig. 4 represent the reported amount of precipitable water vapour (reduced to a 1 km path in the case of Gebbie and co-authors).

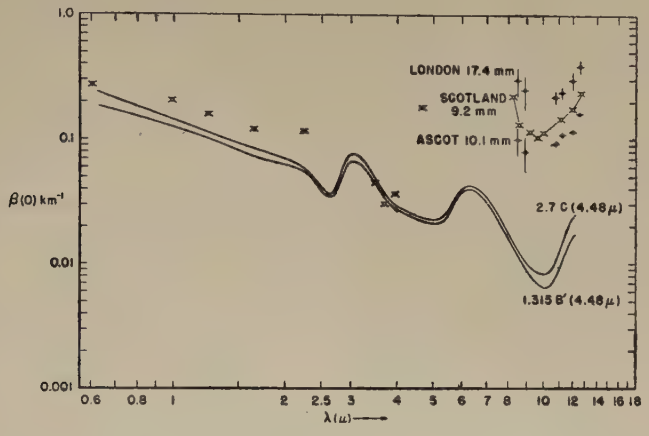


Figure. 4. Comparison of computed extinction with measurements of window extinction in Great Britain.

The continuous curves in Fig. 4 show two of the haze models here considered, adjusted so that they agree with the Gebbie data at λ 0·6 μ and at 3·5 μ. The adjustment is accomplished by means of a constant factor at all wavelengths, equivalent to increasing the original particle concentration by the same factor. The continental model C (4·48 μ) with about 6,200 particles per cm^3, seems to give the best fit with respect to the above-mentioned wavelengths. This seems like a rather turbid condition, but according to the model distribution, 68 per cent of the total number of particles have a radius smaller than 0·10 μ, being in the range of Aitken nuclei and large ions. The other curve shown on Fig. 4, corresponds to a model B′ distribution with 470 particles per cm^3, only 38 per cent of which are in the Aitken range. A better choice of model could be made if observations of the spectral transmission in the visible up to the near ultra-violet were made simultaneously with the infra-red measurements.

This kind of detailed analysis of the haze effect helps in answering some important questions in the observed atmospheric transmission of infra-red. The following points may be made, within the context of our assumptions :

(a) The shape of the pure haze-extinction curve is determined. This allows a more reliable analysis of the effect of ' wing ' absorption, in the so-called window regions. The haze extinction shows window-like regions, one centred around λ 2·6 μ, another between 3·9 and 5·2 μ, and the ' clearest ' window in the region 8 to 12 μ.

(b) In the real, cloudless atmosphere, a narrow region around 3·7 μ seems to have the highest absolute transmission, both as regards molecular absorption by CO_2 and H_2O, and haze scattering.

(c) The observed transmission in the 8 to 12 μ region cannot be attributed to haze alone. Thus, the conclusion reached by Roach and Goody (1958), that the continuous extinction here is mainly due to the wings of water-vapour absorption lines is supported. The effective pure absorption coefficient (assumed exponential) amounts to about 0·10 per gram of water vapour. Roach and Goody estimate that 6,000 3 μ droplets per litre of air are needed to explain this extinction in terms of scattering and they rightly reject this possibility as unrealistic. Their argument is further strengthened if one considers that such a large number of big droplets must necessarily be accompanied by even greater numbers of smaller haze particles. This follows from the experience that small haze particles increase in numbers with relative humidity (cf. Gebbie *et al.* 1951; Deirmendjian 1959b). Thus, if we try to fit the extinction at λ 10 μ with a model A (4·48) distribution (see Table 1) we must increase the concentration threefold, which amounts to a total of 820 particles per cm^3, only 0·5 of which (or 500 per litre) are larger than 3 μ in radius. However, such a haze would result in too great an extinction (about 1·0 per km) at λ 0·6 μ, which is inconsistent with the visual range measurements. Attempts to reproduce the window extinction spectrum by combinations of haze models plus large droplets of a given size also fail. The remarkable agreement of the Ascot measurements with those of the Gebbie group, showing comparable amounts of water vapour along the respective paths (see Fig. 4), suggests that similar conditions of haziness and hence of visual range, must have obtained in each case.

It is regrettable that Roach and Goody did not secure parallel records of extinction in the visible during their Ascot and London measurements, for then the presence of small haze particles would be easily verified. In any case, both theoretical and observational evidence strongly supports the conclusion that line wing absorption is predominant in the atmospheric extinction of λ 10 μ radiation under 'normal' clear sky conditions. Further confirmation of this may also be found in a recent paper by Vigroux (1959), who by an analysis of continuous emission in the 9·6 μ region, independently reaches the same conclusion that water-vapour wings account for an absorption coefficient of 0·10.

(d) In the windows at shorter wavelengths, centred around λ 1·1, 1·25, 1·65, 2·25, and 3·5 μ, scattering on haze particles does account for most of the background extinction. However, here too there may be some wing absorption, as may be seen on Fig. 4. The effect is not as large as is in the 10 μ region, and the absorption coefficient (except at λ 3·5 μ) should amount to 0·05 per gm of water vapour in these windows. The relative magnitude of this absorption with respect to the overall extinction decreases with the wavelength, as should be expected.

Elder and Strong's analysis (1953), in which more or less empirical descriptions of the window transmission are given, may now be put on a firmer physical basis. Their original idea of separating the attenuation into a continuum attributed to scattering and a selective absorption is useful, except that another continuum, due to wing absorption, must also be properly considered. The empirical formula proposed by Elder and Strong (1953, p. 203) is equivalent to an exponential absorption coefficient of only about 0·02 per gm, uniformly for all the windows. Our analysis shows that this may be as much as 0·05 at the centre of windows in the 1 to 3 μ region, and 0·10 in the 10 μ region. Further this type of absorption is not a constant for any one window but may increase on either side of the centre. Some of this increase, but not all, may be due to scattering, as can be judged by comparing the observed and computed curves in the 8 to 12 μ window in Fig. 4. A more detailed discussion of the wing effect in terms of the molecular spectra and models of line shape and spacing is beyond the scope of this paper.

5. Extinction by clouds

In order to get an idea of the extinction by a cloud of water droplets, we consider a single simplified model, roughly corresponding to a tenuous altocumulus of the type accompanied by coloured coronas around the sun or moon. A 'biparabolic' distribution was assumed, of the form

$$n(a) = \frac{n(a_c)}{(a_c - a_1)^4}(a - a_1)^2 (a_2 - a)^2; \quad a_1 \leqq a \leqq a_2 = 2a_c - a_1 . \qquad (13)$$

where $a_1 = 1\ \mu$ and $a_2 = 6\ \mu$ are the smallest and largest droplet radii and $a_c = 3{\cdot}5\ \mu$ is the radius with the largest concentration, $n(a_c)$. This is adjusted so that there is a total of 100 droplets per cm^3. The cloud distribution added to the model A haze type results in the curve labelled ' A + cloud ' on Fig. 2. The number of cloud droplets has been purposely made small in order that the assumption of independent or incoherent scattering may hold.

The extinction coefficients per km of path within such a cloud (excluding water-vapour absorption) are also shown under Table 1 and on Fig. 3, by the curve labelled ' Cl (ext.).' Note that the cloud extinction is an order of magnitude larger than that of the haziest model A (4·48). The transmission per km of any infra-red radiation through such a cloud would be nil, even disregarding any vapour absorption. The only relative window, around 10 μ, will be reduced when the wing absorption is added.

The curve labelled ' A (4·48) + $\frac{1}{10}$ Cl ' on Fig. 3 illustrates the normal optical thickness of such a cloud only 100 meters thick (corresponding to 10^6 droplets per cm^2 column, however distributed vertically) assuming also some haze below, according to the model indicated. The optical thickness in this case is nearly constant and equal to about 1·0 everywhere except around 10 μ, where the wing absorption must be added before estimating the actual transmission.

On account of the presence of infra-red absorption bands in the spectrum of liquid water, the extinction by water droplets is not entirely the result of scattering but it includes some real absorption within the droplets. The relative importance of this absorption changes with the size of the particles relative to the wavelength, as may be seen from the dotted curve, labelled ' abs.' on Fig. 1 (a). The monochromatic absorption cross-section as a function of the size of the droplet was estimated by means of van de Hulst's approximation

$$K(\text{abs.}) = 1 + \frac{e^{-2x}}{x} + \frac{e^{-2x} - 1}{2x^2}, \qquad (14)$$

multiplied by the factors $(1 + D)$ described under Section 2, where x has the same meaning as before. The integration

$$\gamma_\lambda = \int \pi a^2 K'(\text{abs.})\, n(a)\, da \qquad (15)$$

then represents the *absorption coefficient* per unit horizontal path, i.e., the amount actually absorbed within the water droplets. In general this is not equivalent to the bulk absorption $\gamma_w s$, where s is the depth of the layer of water corresponding to the mass of water droplets in the column. Compare the following values :

	Haze Model C (4·48)		Cloud Model	
	λ 6·04 μ	14 μ	6·04 μ	14 μ
γ_λ, km^{-1} (droplets)	0·0095	0·0110	3·66	4·03
$\gamma_w s$ (equiv. water layer)	0·0110	0·0123	6·25	7·04

It is seen that the difference is large in the cloud case. The reason for this is that in large droplets about one half of the incident radiation is diffracted without penetrating the drops. In small droplets, on the other hand, the total extinction cross-section is mostly accounted for by absorption (see Fig. 1a).

The difference $\beta_\lambda - \gamma_\lambda$ represents the attenuation coefficient corresponding to pure scattering. This quantity is indicated by the dash-dot curve labelled ' Cl (scat) ' on

Fig. 3. The fraction of the total extinction coefficient which corresponds to absorption shown under Table 2, for two models and various wavelengths.

TABLE 2. FRACTION OF EXTINCTION COEFFICIENT DUE TO ABSORPTION, $\gamma_\lambda/\beta_\lambda$

λ	3	3·5	5	6·04	8·6	10	12	14 μ
Cloud model	0·35	0·094	0·075	0·37	0·24	0·41	0·67	0·59
Model C (4·48)	0·31	0·072	0·13	0·63	0·42	0·68	0·84	0·82

The quantity $(1 - \gamma_\lambda/\beta_\lambda)$ is the *albedo per unit volume* of space containing a sampl of the scattering particles, and it is an important parameter in estimating the diffus transmission and reflection in clouds of semi-absorbing particles.

Although the corrected van de Hulst approximation still underestimates the absorption by as much as 15 per cent in regions of strong absorption and small relative size, thes results suggest that haze particles and especially large cloud droplets scatter considerabl amounts of the incident infra-red radiation. The amount scattered at each wavelengt will depend on the particle distribution and on the optical depth of the layer considered Large haze particles and tenuous clouds should produce an aureole around the sun in th regions λ 1·1, 1·25, 1·65, 2·25 and 3·5 μ. No aureole should be detectable in a clear sky a 10 μ because of the small relative size of the particles (no diffraction), the low scatterin coefficient, and the weak solar radiation. None, in fact, was detected by Roach an Goody (1958).

However, especially with clouds, there will be some back scattering of terrestriall emitted infra-red in the 10 μ region. The back-scattered radiation must exist in additio to the thermal emission of the water vapour, and its magnitude must depend on the surfac temperature. A cloud cover thus may act as a partial reflector of 10 μ radiation, which i then ' trapped ' in the layer between the ground and the cloud. This mechanism must b important both as regards the radiation balance at the ground and the formation, persistenc and dissipation of clouds. It is a *diabatic* process which is usually neglected in considerin the physics of clouds. It probably plays an important role in the formation of larg tropical disturbances over a warm ocean.

The problem of the apparent infra-red emission from a cloud containing wate droplets and water vapour needs further study. Certain approximations may be vali in the case of very tenuous or very deep clouds, but a correct analysis in the intermediat case and in the case of dense clouds, depends on a rigorous solution of the problem c radiative transfer in such media. Such work is important in estimating the energy budge of large portions of the atmosphere as well as of the entire atmosphere. The infra-re photometry of the atmosphere from space cannot, in itself, provide this information.

A similar problem arises in the spectrophotometric investigation of other sunli planetary atmospheres. If such atmospheres contain elements in various phases – mole cules, minute droplets and crystals – the observed radiations will be a result of absorptio emission *and* scattering, and any apparent emission or absorption bands cannot be simpl related to the depth and temperature of the gas phase alone. Detailed theoretical investiga tions of several model atmospheres will provide a firm basis for the interpretation of obser vations which can be easily made from the earth or from earth-bound instrument carrier

ACKNOWLEDGMENTS

The author is indebted to Mrs. Patricia A. Walters for the laborious calculation c extinction cross-sections by means of a desk computer, and to Mr. William Viezee wh assisted in the cloud-model computation.

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Note added in proof : The result on differences in the absorption by droplets and by an equivalent water layer seems to be borne out by experimental work reported in a paper by Bocharov (1958), which came to my attention after completion of the present paper. *Author.*

551.521.32 : 551.524.35 : 551.524.4 : 551.551.8 : 551.575.1

Measured radiative flux divergence near the ground at night

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(Manuscript received 3 November 1959)

SUMMARY

Nocturnal radiative flux divergence profiles in the lowest few metres have been directly measured and typical cases are presented in graphical form.

Comparison of flux divergence calculated from radiation charts with measured divergence between 0·5 m and 1·5 m for a wide range of wind speeds and cloud conditions has shown the measured value to be up to 2·8 times larger than the calculated one. This discrepancy and others are attributed to the presence of haze.

Since radiative cooling rates were generally found to be higher than actual rates, development of the nocturnal inversion near the surface is shown to be predominantly caused by radiation, with eddy cooling supplanted in the lowest layers by eddy warming.

Periods of radiative warming sometimes associated with long-period (40 min) air-temperature fluctuations were found. A possible explanation is given for these and for fog formation in airlayers not in direct contact with the surface.

Comparisons are given between radiative ground temperatures and those obtained from standard grass thermometers.

1. INTRODUCTION

Knowledge of the long-wave radiative flux divergence, div R, near the ground is important for a wide range of problems associated with microclimatology, agricultural science, and the like, since it determines the rate of radiative cooling or warming of the air. This knowledge is thus very important for problems associated with frost protection and it can shed light on the development of the nocturnal inversion.

That radiation could be a very important factor in airlayers near the ground has been pointed out by Robinson (1950), Rider and Robinson (1951) on partly theoretical grounds, and calculations by Möller (1955) can be interpreted in the same way. Even before this, Johnson (1929) and others had come upon phenomena, mainly sudden cooling, which they had difficulty in explaining by other means and which they tentatively explained as an effect of radiation. Hitherto it has been customary to estimate the radiative flux divergence either from radiation charts such as those of Elsasser (1942), Möller (1943), Yamamoto (1952), and Deacon (1950) or by means of a direct method due to D. L. Brooks (1950) using temperatures and humidities in the lowest 100 metres. Direct measurement was difficult, as net radiometers were neither sensitive nor stable enough to measure small differences with sufficient accuracy.

2. METHOD OF OBSERVATION

An improved polythene-shielded net radiometer (Funk 1959) has been developed in the C.S.I.R.O. Division of Meteorological Physics. It uses polythene hemispheres of only 0·05 mm thickness and a thermopile of 250 junctions. The sensitivity of this instrument is almost an order of magnitude higher than that of previously reported net radiometers. It has a response time of about a minute and a wind dependence of less than 1 per cent over a windspeed range of 0-15 m sec^{-1}.

Two of these instruments were mounted side by side at a certain height either over closely-cut level grassland at the Division's site at Aspendale, or later over pasture at Edithvale, thus being subjected to the same radiative flux R. The two instruments were connected in opposition and the more sensitive one shunted to give a null reading on the galvanometer, thus equalizing the two sensitivities. One of the radiometers was next moved to another height. The potentiometrically measured output e.m.f. is then a measure

Figure 1. Mounting of instruments.

of the flux difference between the two heights and the flux divergence is simply obtained by dividing this difference by the difference in height. Later, both instruments were mounted 1 m vertically apart on a hoist, raised up to 8 m and then lowered to 0·3 m.

The balance was checked at intervals, and the shunt readjusted or the resulting zero drift taken into account. The balance was never found to vary significantly.

On a few occasions measurements were taken in a manner similar to the above, with two radiometers converted to effective pyranometers by shielding their upper hemispheres with constant-temperature enclosures blackened on the inside. This permitted the separate measurement of the *upward* flux divergence (div $R_{\uparrow}$).

The effective radiation surface temperature T_S was also measured with such a pyranometer at a height of 0·5 m using the simple relationship between T_S and the enclosure temperature T_e $\sigma (T_S^4 - T_e^4) =$ constant x e.m.f., the constant being determined from black-body experiments. (The symbols used are standard, unless otherwise stated). Radiation temperatures determined in this way are more representative than those obtained by pointing an actinometer downwards, since they take the full hemisphere properly into account. The readings were corrected for the small influence of the water vapour between the instruments and the ground.

The measured flux at 1·5 m height was found to be independent of position when an instrument was rotated horizontally with a radius of 1·2 m. Since an instrument at that height receives about 90 per cent of its upward flux from a surface area 4·5 m in radius it was concluded that in order to avoid sampling errors and to eliminate effects of a simulated flux divergence due to inhomogeneities of the surface, the lower instrument had to scan a similar area. This was accomplished by rotating the instruments round a mast at a distance of 1·2 m (Fig. 1, Plate IV). In the beginning readings were taken in three positions 120° apart and later both instruments were continuously rotated whilst the readings were taken. The 1·5 m instrument need not have been, but for simplicity was rotated also. Observations were taken on four different sites at Aspendale and three at Edithvale, thus reducing sampling errors still further.

Since the method used in determining the total flux divergence is a true difference method, readings were not influenced by variations in total flux, no matter how rapid, and it was therefore possible to take readings under a wide range of cloud conditions. In assessing the accuracy of the readings, account was taken of unbalance, wind shear, and differing obstruction of the fields of view of the two radiometers due to the mounting. The possible error amounts to about 0·1 μw cm^{-3} for div R, and for div $R_{\uparrow}$ a rather larger value (0·15 μw cm^{-3}) owing to increased zero drift caused by differences in enclosure temperatures.

To provide a comparison of the measurements with radiation charts, temperature and humidity profiles were taken in the lowest 10 m. In addition, two ordinary mercury thermometers were used to measure the surface temperature, one placed on top of the grass and one inserted into the foliage. These temperatures were used only for comparison with the radiation temperature. The latter was always used in plotting the radiation chart.

A comparison of a grass minimum thermometer (which apparently indicated the radiation temperature when used by Robinson (1947)), with a mercury thermometer placed side by side on top of the grass was also carried out but no real difference could be found.

3. Observational results

So far measurements have been carried out at various times on 39 nights. After initial trials Deacon's chart was used for comparison since it was found to be in better agreement with the measurements and more convenient to use. Judging from the literature Robinson (1950) the Kew chart would have given similar results, whilst Elsasser's chart gave values lower than Deacon's by about 20 per cent.

A representative selection of these results is given in Table 1.

TABLE 1. VALUES OF OBSERVED AND CALCULATED RADIATIVE FLUX DIVERGENCE

Date 1959	Time (EST)	Radiative flux R at 1·5 m (mw cm)$^{-2}$	Radiative flux divergence div R (μw cm^{-3})	Calculated div R (μw cm^{-3})	Observed / Calculated	Radiative cooling rate (°C hr^{-1})	U_1 (m sec^{-1})	Clouds
24 April	1910	− 2·8	1·5	0·7	2·1	4·2	N. 4·0	8/8 Ac
29 April	1835	− 7·9	1·5	0·8	1·9	4·2	N. 3·0	Nil
29 April	1915	− 7·5	2·0	1·0	2·0	5·5	N. 3·0	Nil
5 May	1827	− 3·9	1·1	0·6	1·9	3·0	SW. 4·0	3/8 Ac
5 May	1850	− 4·9	1·2	0·6	2·1	3·3	SW. 4·0	3/8 Ac
7 May	1800	− 6·7	1·4	1·0	1·4	3·9	SSE. 1·1	Nil
7 May	1825	− 6·4	1·7	0·9	1·9	4·7	SSE. 1·1	Nil
8 May	1800	− 8·1	1·6	0·9	1·8	4·4	WSW. 1·7	Tr Cu
14 May	1755	− 4·2	4·4	2·3	1·9	12·2	SE. 0·5	Nil
14 May	1825	− 4·1	2·8	1·7	1·7	7·8	SE. 0·5	Nil
18 May	1800	− 1·5	1·2	0·8	1·5	3·3	NE. 0·8	8/8 Cu Sc
19 May	1805	− 2·8	0·5	0·4	1·3	1·4	SW. 4·0	5/8 Cu Cb
19 May	1845	− 2·9	0·7	0·5	1·4	1·9	SW. 4·0	5/8 Cu Cb
28 May	1805	− 4·2	0·5	0·3	1·6	1·4	SW. 2·0	Tr Ac As
28 May	1850	− 10·2	1·1	0·4	2·8	3·0	SW. 2·0	Tr Ac As
29 May	1840	− 6·3	0·7	0·4	1·7	1·9	S. 2·6	3/8 Cu Ac
1 June	1820	− 4·9	1·9	0·8	2·4	5·3	ESE. 0·5	2/8 Ac
1 June	1850	− 4·9	1·5	0·8	2·0	4·2	ESE. 0·5	2/8 Ac
4 June	0528	− 3·9	0·5	0·3	1·7	1·4	ENE. 0·6	7/8 Ac
4 June	0630	− 3·1	0	0·1	−	0	ENE. 0·6	7/8 Ac
4 June	0705	− 2·3	− 0·1	0·08	−	− 0·3	E. 0·6	7/8 Ac
16 June	1745	− 8·9	1·5	0·7	2·2	4·2	N. 2·2	Nil
22 June	1740	− 3·8	1·6*	1·0*	1·6	−	S. 6·0	8/8 Cu
6 July	1750	− 2·3	0·33‡	0·2‡	1·7	0·9	N. 3·0	6/8 Ac As
6 July	1830	−	0·45*‡	0·32*‡	1·4	−	N. 3·0	8/8 Ac As
6 July	1840	−	0·64*†	0·33*†	1·9	−	N. 3·0	8/8 Ac As

* div $R_{\uparrow}$. † average 0·2-2 m. All others average 0·5-1·5 m. ‡ average 1-2 m.

Radiative cooling rates as high as 12°C hr^{-1} were measured. No mist was observed on any occasion where comparisons with the chart were carried out.

Under the wide range of conditions and magnitudes covered by these results the measured div R (also div $R_{\uparrow}$) is consistently considerably higher than the computed value, the ratio varying from 1·3 to 2·8. This ratio may be somewhat inaccurate for low divergence values since although the radiation ground temperature was measured within 0·1°C it could not be measured simultaneously with the divergence. The necessary interpolation may have caused an error of up to 0·2°C introducing a maximum error of 0·16 μw cm^{-3} into the calculation.

By neglecting advection a comparison of radiative and actual cooling rates can be used to determine div $H \approx \dfrac{\partial H}{\partial z}$ the divergence of eddy heat flux.

$$\frac{\partial H}{\partial z} = c_p \rho (C_A - C_R) \quad . \quad . \quad . \quad . \quad (1)$$

where

$$C_R = \frac{1}{c_p \rho} \text{div } R \quad . \quad . \quad . \quad . \quad . \quad (2)$$

is the radiative cooling rate and

$$C_A = -\frac{\partial T}{\partial t} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

the actual cooling rate.

Table 2 shows that at Aspendale on all calm evenings C_R was higher than C_A when eraged over more than an hour. This was also mostly the case at Edithvale despite ccasional high instantaneous actual cooling rates (Figs. 2-5), and shows that the layers question were in effect being mostly warmed by eddy transfer of heat. The actual agnitude of div H would depend on whether advection can be neglected.

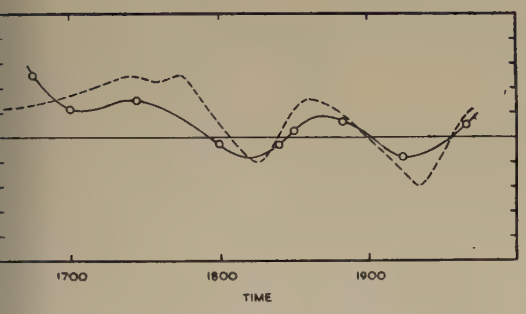


Figure 2. 10 July.

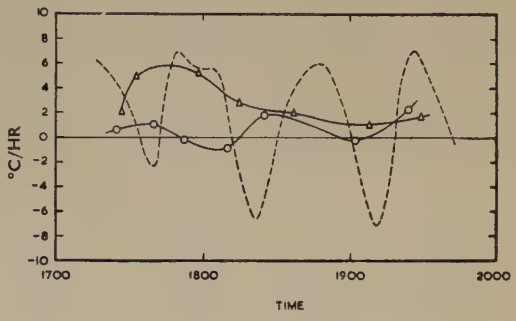
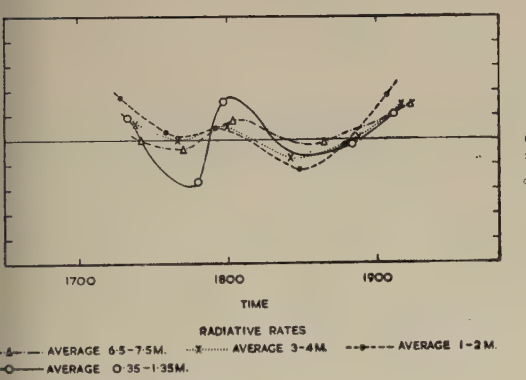


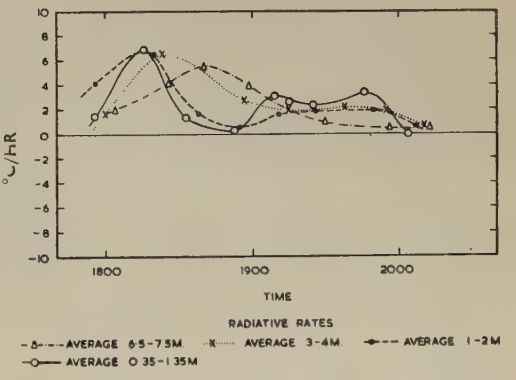
Figure 3. 17 July.

– – – – – Actual rate 1·5 m
O——O Radiative rate, average 1-2 m

– – – – – Actual rate 2 m
O——O Radiative rate, average 1-2 m
△——△ Radiative rate, average 2-4 m

res 2 and 3. Radiative (C_R) and actual (C_A) cooling rates at Edithvale on the evenings of 10 and 17 July 1959.





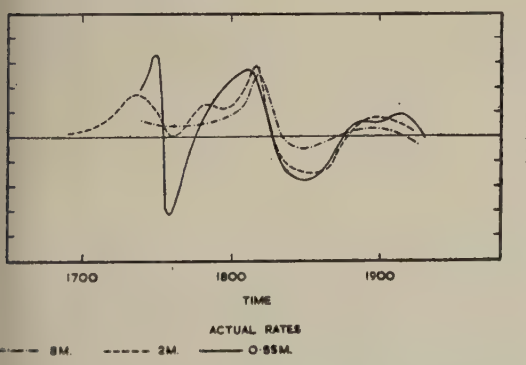


Figure 4. 21 July.

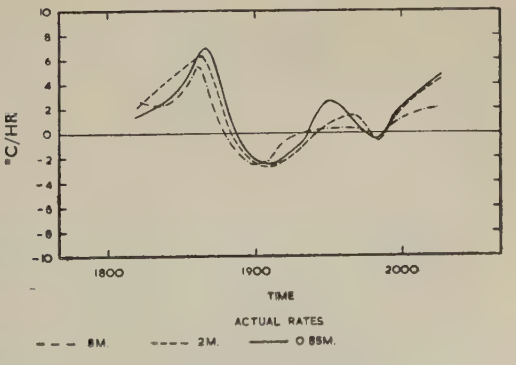


Figure 5. 31 July.

res 4 and 5. Radiative (C_R) and actual (C_A) cooling rates at Edithvale on the evenings of 21 and 31 July 1959.

TABLE 2. MEAN RADIATIVE AND ACTUAL COOLING RATES AND EDDY HEAT FLUX DIVERGENCE (NEGLECTING ADVECTION) ON CALM NIGHTS

Date 1959	Time interval (EST)	Radiative cooling rate C_R (°C hr^{-1})	Observed cooling rate C_A at 1·5 m (°C hr^{-1})	Divergence of eddy heat flux $c_p \rho (C_A - C_R)$ (μw cm^{-3})	Lapse rate (°C m^{-1})	U_1 (m sec^{-1})	Site
7 May	1740-1840	4·3*	1·1	− 1·2	− 0·3*	SSE. 1·1	Aspendale e
14 May	1738-1838	9·9*	3·3	− 2·3	− 0·7*	SE. 0·5	Aspendale e
1 June	1750-1850	5·0*	0·7	− 1·6	− 1·9*	ESE. 0·5	Aspendale e
4 June	0538-0708	0·4*	0·0	− 0·1	− 0·3*	E. 0·6	Aspendale e
10 July	1645-1940	0·8†	2·3	+ 0·5	− 1·6†	mist N. 0·5	Edithvale
17 July	1725-1925	1·6†	1·1	− 0·07	− 1·4†	SW. 0·5	Edithvale
21 July	1730-1938	0·7†	0·7	0	− 0·6†	S. 1·0	Edithvale
31 July	1800-2000	2·4†	0·7	− 0·6	− 0·3‡	S. 1·0	Edithvale
25 August	1830-2000	6·6*	1·5	− 1·9	− 0·7*	S. 2·0	Aspendale e

* Average 0·5-1·5 m. † Average 1-2 m. ‡ Average 0·75-2 m.

It should be pointed out that, apart from one morning run, these readings were limited to the earlier hours of the night.*

Periodic radiative warming was observed on all sufficiently calm nights when readings were taken with improved instrumentation permitting readings at much shorter intervals. This was often in phase with actual warming, the phase being sometimes maintained over even more than one cycle.

TABLE 3. RADIATION AND MEASURED SURFACE TEMPERATURES

Date 1959	Time (EST)	Radiation Surface temperature T_S (°C)	Surface temperature measured with mercury thermometer on top of grass (°C)	Surface temperature measured with mercury thermometer inserted into foliage (°C)	U_1 (m sec^{-1})	Clouds
5 May	1751	15·7	15·4	15·7	SW. 4·0	3/8 Ac
7 May	1840	9·9	8·8	10·7	SSE. 1·1	Nil
8 May	1832	10·8	10·0	11·8	WSW. 1·7	Tr Cu
11 May	1745	3·7	2·2	9·7	E. 0·5	Nil
14 May	1930	1·1	− 1·1	3·1	SE. 0·5	Nil
18 May	1732	13·6	12·7	13·7	NE. 0·8	8/8 Cu Sc
19 May	1850	13·1	12·4	13·3	SW. 4·0	5/8 Cu Cb
28 May	1915	10·8	11·1	10·6	SW. 2·0	Tr Ac As
4 June	0645	4·3	3·6	5·4	ENE. 0·6	7/8 Ac

* In spite of the well established low emissivity of polythene film, dew seems to form almost as readily on the hemispheres as elsewhere. Whether this is an indirect effect of the strong surface charge normally present on polythene is under investigation. Heating rings, normally employed to prevent dew formation on the hemispheres, cannot be used during or just before any actual measurements, owing to the smallness of the radiation differences involved. Thus if, as often happens later at night, the time taken for dew to form after heating has ceased becomes comparable with the response time of the instrument, accurate readings are no longer possible.

Figs. 2, 3, 4 and 5 show this phenomenon in more detail for varying levels from 0·35 to 7·5 m. It is noteworthy that on several occasions the shape of the C_R curve (smoothed to eliminate short-term variations) at a particular level sometimes closely resembles that of C_A at the same level (viz. Figs. 2 and 4). The period of these fluctuations was roughly 40 mins. Periodic radiation ground-temperature variations were also frequently observed.

We see from Table 3 that T_S lies between the temperatures indicated by the two thermometers. The temperature on top of the grass was generally lower than that in the foliage. The one exception was on 28 May 1959 when, just after light rain, the order was reversed.

4. Discussion

(*a*) *Disagreement between measurements and chart*

By comparison with measurements, charts were found to be qualitatively correct as far as mean values over periods of more than an hour were concerned and for instantaneous values on most nights without a marked surface inversion. However, quite often considerable discrepancies were observed in particular, neither the measured radiative warming nor most of the div R profiles on calm nights (Figs. 2-5) could have been deduced from the observed temperature and humidity profiles, with no observed mist or fog.

It is apparent from the measurements that radiation charts systematically and considerably underestimate div R. Two possible causes of this discrepancy suggest themselves.

In principle the factor for conversions from column to slab emissivities as used in the charts (1·78 for Deacon's) could be wrong, owing to the insufficiently known fine structure of the water-vapour absorption bands, although hardly by a sufficient amount to account for the whole effect.

All these comparisons were carried out on nights free of mist, while the wide range of wind speeds rules out abnormal concentrations of absorbing gases or other matter. On the other hand, the swelling of normally present nuclei, both natural and artificial, could affect absorption materially even before mist or fog develops. Such an effect would not be detected in laboratory measurements such as those of F. A. Brooks (1941) owing to the comparatively dry air used.

The most likely explanation of the inadequacy of radiation charts, at least under nocturnal conditions, is therefore, their neglect of haze in the air near the surface, as previously suggested by Robinson (1950). The strongly layered structure indicated by the measured div R profiles (cf. Figs. 3 and 4) reinforces this conclusion.

No statistically significant correlation between relative humidity and the ratio observed : calculated div R has been found. This is not surprising since, assuming haze to be present, radiative transfer would, in general, depend on its vertical distribution*, so far unknown, as well as on the temperature and humidity profiles, with only the latter taken into account by the chart. Moreover, the relationship between haze and relative humidity is a complicated one, depending on other parameters as well.

(*b*) *Nocturnal inversion*

Since, as is seen from Table 2, in the lowest few meters radiative cooling rates were mostly found to be higher than actual rates, the development of the nocturnal inversion near the ground is generally driven by radiation, with turbulent transfer often acting as a brake. It is further seen that the average radiative cooling rates observed at Aspendale were generally higher than those at Edithvale under similar conditions. Whether this is an environmental effect or a property of the different plant-cover and soil-moisture content in the two localities is under investigation.

* Referred to by Stewart (1955) as 'detailed structure of radiative properties over the site.'

(c) *Long-period temperature fluctuations*

Air temperature fluctuations of about 40 min period which have also been observed elsewhere (Johnson 1929; Raschke 1957) appear to be a regular feature of nocturnal inversions. Radiative transfer probably plays an important part in this phenomenon too,* since both the magnitude and periodicity of the radiative warming or cooling are of the right magnitude. In principle, an in-phase relationship between C_R and C_A would point to div R being a major contributing factor to the observed instantaneous C_A. An out-of-phase or more random relationship could be attributed to influences such as advection from a neighbouring area or, as suggested by Johnson (1929), changes in wind speed: other evidence gained during these experiments tends to favour the first of these alternatives. In no case was a breakdown of the inversion at these heights observed. Horizontal air temperature differences at 1-2 m heights over distances of some tens of metres and of similar periodicity observed both at Edithvale and by Best (1931) fit well into the picture. On one occasion temporary warming and cooling in antiphase were observed at two points only 40 m apart.

Some of these air temperature fluctuations may be initiated by the previously mentioned changes of the ground temperature (actually T_S) possibly caused by a periodic breakdown of the strong inversion in the lowest few centimetres of the air.

(d) *Radiation fog formation*

Emmons and Montgomery (1947) proposed that radiative cooling of the air itself as well as surface cooling is generally essential for radiation fog formation as, apart from second-order effects, turbulent loss of moisture at a given level will be too rapid in relation to that of heat from a nearly saturated atmosphere to a colder surface. From theoretical radiation considerations, Fleagle (1953) finds radiative warming immediately above the surface and concludes that fog formation should generally not occur in air-layers directly in contact with a cold surface, but at heights near a maximum of C_R.

Both the vertical C_R distributions (Figs. 3 and 4), which show layers of radiative warming or reduced cooling not restricted to the immediate vicinity of the surface, and the fact that in the air-layers investigated cooling was either completely or predominantly caused by radiation and not be turbulence, lends further support to Fleagle's conclusions. However, conditions may well be considerably more complicated than those he assumed.

The suggestion of complex C_R distributions in the present work can explain the formation of multiple layers of mist or fog some metres above the surface in addition to single ones. At the Edithvale site mist or fog formation in layers above the surface has frequently been observed, the 30 m tower with its lights assisting such visual observations. Furthermore, on at least three occasions, two separate layers were seen.

5. Conclusions

It appears to be established from the measurements that radiative heat exchange plays an important part in the heat budget in the layers near the ground.

Since both the temperature profiles and actual cooling rates were much the same as observed elsewhere and since the average radiative cooling rate was practically never found to be lower than the actual one, we can conclude that in the lowest few metres as a rule :

(1) night-time cooling is predominantly of radiative origin, and

(2) that at certain times, as predicted by Rider and Robinson (1951), these lowest air-layers tend to be *warmed* by divergence of eddy transfer of heat.

* On some of these occasions mist formation was observed, but calculation shows that the rate of latent heat release cannot account for the warming observed, quite apart from the fact that haze formation was not correlated with these phenomena.

The observed vertical radiative cooling-rate distributions could explain the formation of both single and multiple fog-layers above a cold surface. These distributions and the underestimation of the radiative flux divergence by the chart can be attributed to an underestimation of the absorbtivity of air. This is most likely to be due to haze. Neglect of haze effects makes the use of charts of doubful value near the ground and may largely explain serious errors suspected by Stewart (1955) and others. The presence of absorbers not normally encountered can be ruled out.

Over a typical grass surface at Aspendale radiation temperatures could not be measured reliably with single thermometers, no matter how placed, but readings from two thermometers, one on top of the grass and one inserted into the foliage, invariably bracketed the radiation temperature.

Acknowledgment

I am indebted to many members of the C.S.I.R.O. Division of Meteorological Physics for valuable advice. Mr. C. L. Garnham took most of the measurements.

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The diabatic wind profile

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(Manuscript received 31 March 1959; in revised form 23 December 1959)

SUMMARY

Wind profiles observed at various locations are compared with wind profiles predicted by Ellison on the basis of similarity theory. It is shown that theory and observations agree well for near-neutral and unstable air; however, in stable air, factors not considered in the similarity theory may become important.

The range of usefulness of the log-linear wind profile is shown to be small.

1. INTRODUCTION

Monin and Obukhov (1954) have shown from similarity theory that there should exist a universal relation between S, defined by $S = \frac{kz}{\overset{*}{u}} \frac{\partial V}{\partial z}$, and $\frac{z}{L}$, where L is defined by $L = \frac{\overset{*}{u}^3 C_p \rho T}{kg H}$.

Here

z = height

V = wind speed

$\overset{*}{u}$ = friction velocity

k = von Kármán's constant

ρ = density

T = temperature

g = gravity

H = vertical flux of heat.

Physically, S can be regarded as a 'normalized' logarithmic wind shear. It has the property that it is unity up to at least 10 m in a neutral atmosphere, and approaches unity close to the ground in any atmosphere. It is related to the 'generalized' von Kármán constant (also called the generalized eddy viscosity) $\overset{*}{K}_m$, by: $S = \frac{k}{\overset{*}{K}_m}$.

For forced convection and for free convection, the relationship between S and $\frac{z}{L}$ is well known. According to Priestley (1955) it is possible to explain the conditions in the surface boundary layer generally by one or the other relation, with a sudden transition between the extremes; on the other hand, Ellison (1957) and Businger (1959) have suggested that interpolation formulae may be useful. The exact character of such interpolation formulae cannot be derived from similarity theory. Businger (1959) determined an interpolation formula from mixing-length considerations; Ellison (1957) found a simpler relationship which was designed just to fit the known relations in free and forced convection at the proper limits. It is the purpose of this paper to compare Ellison's hypothesis with observations.

* Mineral Industries Experiment Station, Contribution No. 59-46.

After some transformation, Ellison's equation can be written :

$$S^4 + \frac{\gamma z}{L} S^3 = 1 \quad . \quad . \quad . \quad . \quad (1)$$

where γ is a constant, to be determined from observations. From this equation, either L (and therefore the vertical heat flux) can be determined from a given wind profile; or, given the roughness length, the heat flux and the friction velocity, the wind profile can be computed by numerical integration.

In many practical situations, the heat flux is not measured. It is therefore practical to introduce a ' gradient length ' L', which is defined by

$$L' = -\frac{\mathring{u}\theta \dfrac{\partial V}{\partial z}}{kg \dfrac{\partial \theta}{\partial z}} \quad . \quad . \quad . \quad . \quad (2)$$

where θ is potential temperature. L' is related to L by :

$$L' = \frac{K_h}{K_m} L$$

Here K_h and K_m are exchange coefficients for momentum and heat. We can then re-state the basic equation :

$$S^4 + \frac{\gamma' z}{L'} S^3 = 1 \quad . \quad . \quad . \quad . \quad (3)$$

Here $\gamma' = \dfrac{K_h}{K_m} \gamma$.

Fig. 1 compares Eq. (3) with observations, with the arbitrary choice of $\gamma' = 18$ to yield the best fit. The solid line represents the theoretical relationship. Each point is

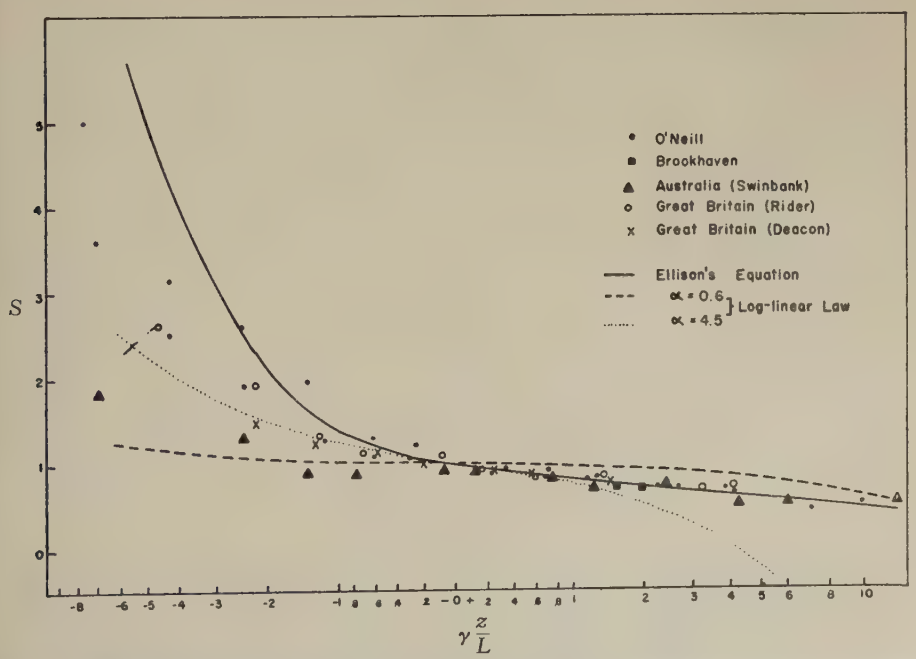

Figure 1. Theoretical relation between S and $\gamma \frac{z}{L}$ with observations. The dashed and dotted curves show the relation according to the Monin-Obukhov law for two values of the constant α

TABLE 1. NUMBER OF WIND PROFILES AVERAGED FOR EACH POINT OF FIG. 1

Range of $\frac{z}{L'}$	− 0·500 − 0·301	− 0·300 − 0·201	− 0·200 − 0·101	− 0·100 − 0·051	− 0·050 − 0·026	− 0·025 − 0·001	0·000 + 0·024	0·025 0·049	0·050 0·099	0·100 0·199	0·200 0·299	0·300 0·499	0·500 0·999
O'Neill, Nebraska, Lettau and Davidson (1957) 3-6 m 4-8 m	7	8	11	19	21	14	13	21	15	6	1		
O'Neill, Nebraska Barad (1958) 1-4 m 4-8 m	4	6	10	11	12	15	6	12	14	11	7	7	2
Melbourne, Australia Swinbank (1955) 2-8 m 8-32 m	2		3	3	2	1	9	9	16	14	6	6	11
Cardington, England Rider (1954) 0·5-1·5 m		1	1	3	3	7	21	5	2	2	3		
Porton, England Deacon (1949) 0·5-4·0 m			4	6	13	12	23	12	15				
Brookhaven, New York (unpublished) July 1950 11-46 m								75	30				

derived from averages for many profiles in the same range of $\frac{z}{L}$. Details concerning the observations are given in Table 1. This Table contains the location of the observations, the source of the data, the height range, and the number of observations averaged in each case.

In order to evaluate the quantities required for Eq. 3, S was approximated from the ratio of velocity differences over a finite height range, and applied at the geometric mean height within this range. u^* was obtained in most cases from the wind at a low level with the aid of the logarithmic profile. In some cases, the lowest observation was not sufficiently low to avoid the effect of buoyancy on the wind; in those cases a correction (based on the full theory) had to be made. Also, actual measurements of the Reynolds stress were used for two sets of observations.

Fig. 1 shows that Eq. 3 fits the observations in unstable air quite well even if γ' is assumed to be constant. This does not prove that γ' really is a constant, but that assuming it to be constant provides a good practical approximation.

On the stable side, however, the theory does not fit all the points. In fact, here S does not seem to be a unique function of $\frac{z}{L'}$, but rather is a different one for each set of observations. This behaviour suggests that the wind profile on the stable side depends on factors which so far have not been included in similarity theory; for example, Davidson and Barad (1956) have shown empirically that the wind shear above the observed layers has a bearing on the wind profile. It is suggested here (and, indeed, implied by Ellison (1957)), that the eddy structure in neutral and unstable stratifications is governed by the distance from the ground, whereas in stable air the eddies have too small a vertical extent to be governed by this factor.

When $\frac{z}{L'}$ is small, Eq. (3) leads to the well known Monin-Obukhov profile (1954) :

$$V = \frac{u^*}{k}\left[\ln\frac{z}{z_0} - \frac{\gamma' z}{4L'}\right]$$

The quantity $-\frac{\gamma'}{4}$ can be interpreted as Monin-Obukhov's ' constant,' α. Since the optimum value of γ' was 18, α should be about 4·5, in contrast to the value of 0·6, suggested by Monin and Obukhov. The log-linear profile with $\alpha = 4{\cdot}5$ has been plotted as a dotted line in Fig. 1. Apparently, it is tangent to the line derived from Ellison's theory near $\frac{z}{L'} = 0$, as required, but it yields a poor fit to most observations. The dashed curve, drawn with $\alpha = 0{\cdot}6$, fits the observations better on the average, but leads to systematic differences, particularly near $\frac{z}{L'} = 0$, as was pointed out by Priestley (1959 p. 29). The log-linear profile is derived, however, on the basis of a Taylor-series expansion, and should be reliable primarily for small $\frac{z}{L}$. Thus, the value $\alpha = 0{\cdot}6$ is theoretically inconsistent with $\gamma' = 18$ but leads to a fair over-all description of the wind profiles. The value $\alpha = 4{\cdot}5$ is consistent with the more general theory but its range of applicability is small.

2. Wind profiles

Fig. 2 serves to illustrate wind profiles implied by Eq. 3, for observations made at O'Neill, Nebraska, in 1956. In each of the four cases, L' was derived from mean wind and temperature profiles. z_0, as obtained from neutral profiles, was 0·65 cm. u^* was determined in such a way that the theoretical profile was forced to pass through the observed average between the 25 cm wind and the 50 cm wind, at a height of 35 cm.

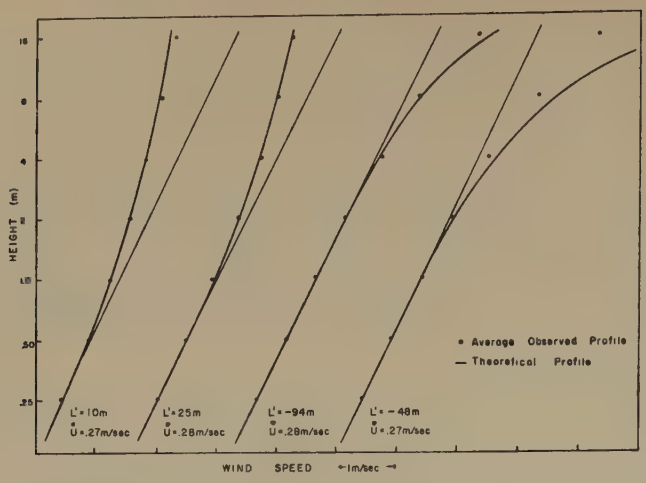

Figure 2. Wind profiles, theoretical and O'Neill, Nebraska observations for four values of L'.

It is clear that the fit is fair. Particularly large errors appear at the higher levels in stable air, as is to be expected from the earlier discussions. Some small systematic errors also appear at the higher levels in unstable air, which could be avoided by a slightly smaller γ'. However, observations elsewhere make a reduction in γ' undesirable. Further, the uncertainties in L' and $\mathring{u}$ are sufficient to throw doubt on the importance of these small errors.

3. Wind shear and Richardson number

From the definitions of L, S and the gradient Richardson number Ri, Eq. (3) can be transformed into

$$S = (1 - \gamma' Ri)^{-\frac{1}{4}} \quad . \quad . \quad . \quad . \quad (4)$$

an equation which was explicitly suggested by Ellison (1957). This can be compared with earlier suggestions (e.g. Holzman 1943)

$$S = (1 - \sigma Ri)^{-\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (5)$$

and Monin and Obukhov (1954) for the limit of small Ri :

$$S = (1 - \alpha Ri)^{-1} \quad . \quad . \quad . \quad . \quad (6)$$

All these equations agree well at small Ri, but only Eq. (4) approaches the correct limit at large Ri. Numerically, agreement of the three equations at small Ri requires that : $\gamma' = 2\sigma = 4\alpha$. Thus it is interesting that $\gamma' = 18$ implies $\sigma = 9$ and $\alpha = 4{\cdot}5$. This should be compared with $\sigma \sim 8$, reported by Priestley (1959 p. 32).

4. Deason's β and Richardson number

A quantity β is frequently defined by : $\beta = -\dfrac{z\dfrac{\partial^2 V}{\partial z^2}}{\dfrac{\partial V}{\partial z}}$. If this quantity were independent of height, the resulting wind profile would be that of Deacon (1949). This definition and Eq. 3 yield :

$$\gamma' Ri = \frac{1 - \beta}{1 - \frac{3}{4}\beta} \quad . \quad . \quad . \quad . \quad (7)$$

Fig. 3 shows a comparison of Eq. 7 with observations collected by Businger (1959), some of which are the same observations made at O'Neill, which were used in earlier figures. Again the results are similar.

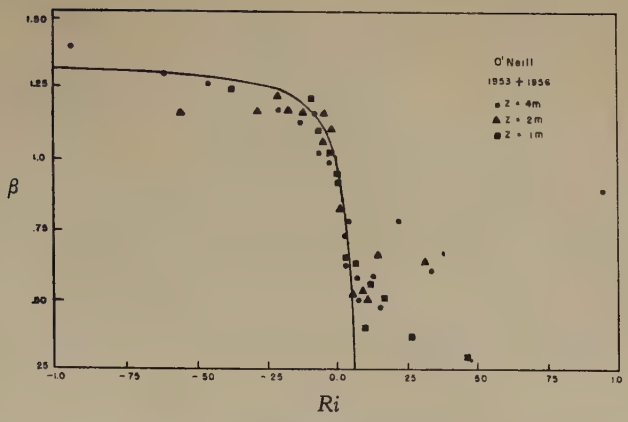

Figure 3. Theoretical and observed relation between Deacon's β and Richardson number. Observations from Businger (1959).

5. The eddy viscosity

The kinematic viscosity is defined by :

$$K_m = \frac{\overset{*}{u}^2}{\frac{\partial V}{\partial z}} \qquad . \quad . \quad . \quad . \quad . \quad (8)$$

From Eq. 1 the following relation can be deduced :

$$K_m = [\epsilon_1 + \gamma \epsilon_2]^{1/3} (kz)^{4/3} \qquad . \quad . \quad . \quad (9)$$

Here ϵ_1 is $\overset{*}{u}^2 \frac{\partial V}{\partial z}$ and measures the rate at which mechanical turbulent energy is produced; the quantity $\epsilon_2 = \frac{gH}{c_p \rho T}$ measures the rate at which buoyancy produces energy.

This can be compared with Heisenberg's expression (1948)

$$K_m \sim \epsilon^{\frac{1}{3}} l^{4/3} \qquad . \quad . \quad . \quad . \quad . \quad (10)$$

Here, ϵ is the rate of energy dissipation, and, for equilibrium, the rate of energy production; l is an eddy scale.

Comparison of Eqs. 9 and 10 makes plausible the identification of the height z as a ' scale ' of turbulence for the situations for which Eq. 9 is valid, namely for unstable and neutral stratification. However, since Eq. 1, and therefore Eq. 9, do not appear to be valid in stable conditions, the height above the ground has no such meaning in stable air.

Eq. (9) also suggests that γ is the relative efficiency of mechanical and convective energy sources in the production of vertical momentum transport.

6. Priestley's ' reduced ' heat flux

The relation between heat flux, eddy coefficient of heat conduction and gradient of potential temperature yields, with the use of Eq. 8 :

$$\overset{*}{H} = \frac{K_h}{K_m} k^2 \left(\gamma' + \frac{1}{Ri}\right)^{1/2} \qquad . \quad . \quad . \quad (11)$$

Here, $\overset{*}{H}$ is Priestley's reduced heat flux (1955), defined by :

$$\overset{*}{H} = \frac{H}{\rho c_p \left(\frac{g}{\theta}\right)^{1/2} \left|\frac{\partial \theta}{\partial z}\right|^{3/2} z^2} \quad . \quad . \quad . \quad (12)$$

As is to be expected, Eq. 12 approaches the proper limits for pure forced, and pure free, convection. Observations of $\overset{*}{H}$ by R. J. Taylor (1956) corrected for high-frequency contributions (Priestley 1959, p. 44) suggest that the numerical value for $\overset{*}{H}$ at the limit for free convection is 0·9. From this and the assumption that γ' is 18, $\frac{K_h}{K_m}$ becomes 1·3, at the convective limit. Consequently γ in Eq. 1 is 14. Of course, these numerical values must be regarded as extremely tentative. All one can really say is that γ and γ' are of order 15.

Unlike the Australia observations, the O'Neill observations do not suggest a sudden transition of $\overset{*}{H}$ between free and forced convection.

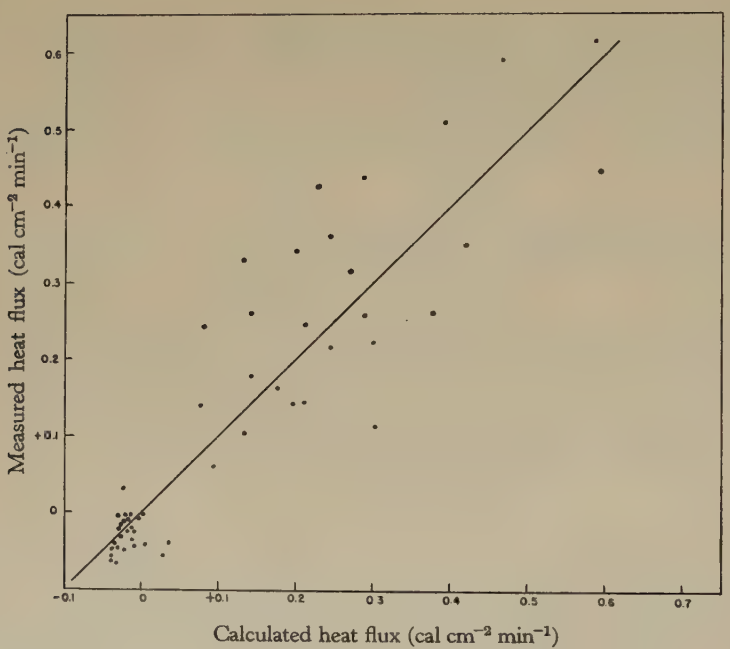

Figure 4. Observed and calculated values of heat flux for 1953 O'Neill, Nebraska observations.

7. Determination of heat flux from wind profile

Given γ Eq. 1 can be solved for the flux of heat. This was done by using the wind observations at O'Neill in 1953, for times when heat fluxes were independently available calculated from the heat balance at the surface (see Lettau and Davidson 1957). γ was assumed to equal 14. Winds at 8 m and 1 m were employed to calculate $\frac{\partial V}{\partial \ln z}$, and $\overset{*}{u}$ was estimated from the wind at 50 cm and the known roughness length. The comparison between observed and computed heat flux is given in Fig. 4. Even though the scatter is large, the agreement between the two techniques of estimating heat flux is good, and Eq. 1 gives satisfactory estimates of heat flux with $\gamma = 14$.

8. Estimation of the exponent of a power law

For engineering applications, it is frequently necessary to approximate the wind profile in a given height range by a power law :

$$\frac{V}{V_1} = \left(\frac{z}{z_1}\right)^p$$

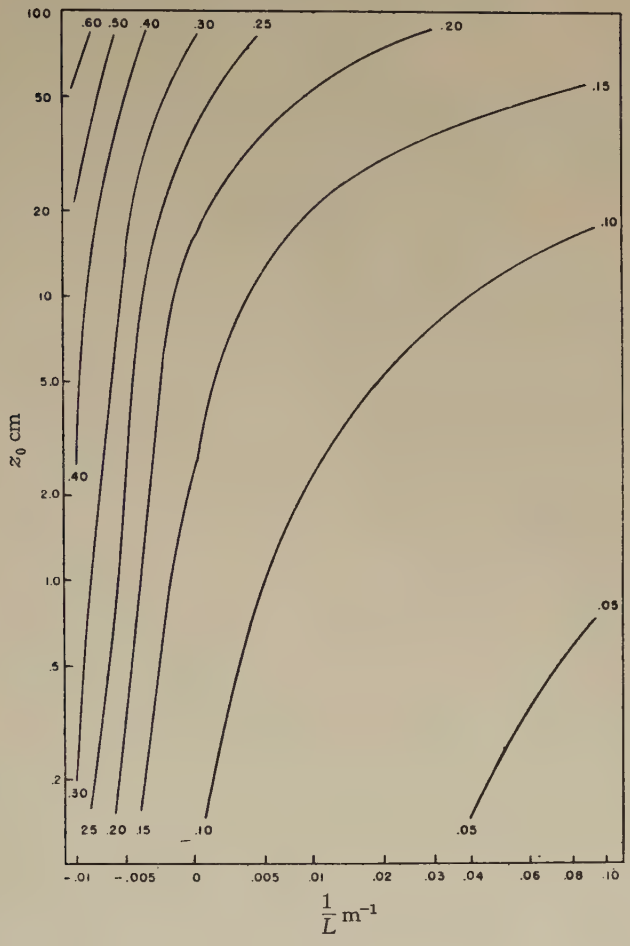

Figure 5. Theoretical variation of the power-law exponent as a function of z_0 and L.

Here V_1 is the wind speed at reference level z_1. p is taken as a constant. Since p is given by $\frac{\partial (\ln V)}{\partial (\ln z)}$, it is also equal to $\frac{\dot{u}S}{kV}$. Therefore, it can be calculated from Eq. 3, given L', z_0, and the height. For example, Fig. 5 shows the dependence of p on L and z_0 for the height range of 11 m to 46 m. The results are in good agreement with observations (see, for example, DeMarrais 1959).

Acknowledgment

The authors are grateful for the support of this research by The Geophysics Research Directorate, Air Force Cambridge Research Center, Air Research and Development Command, United States Air Force, Bedford, Massachusetts.

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551.594.11 : 551.594.13 : 623.454.8

Some recent changes in atmospheric electricity and their cause

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(Manuscript received 7 January 1960)

SUMMARY

Annual mean values of atmospheric electrical potential gradient are tabulated for three British and two Portuguese stations and it is shown that unusual changes have occurred since 1952. These changes could be explained by an increased conductivity of the air near the ground. Measurements of β-ray ionization made at the British stations indicate that the increase in conductivity has been produced by a surface deposit of radioactive material attributable to nuclear-test explosions.

1. Changes in atmospheric electricity

It was pointed out by Pierce (1957) that the fine-weather potential gradient measured at Eskdalemuir showed a striking decrease from about 1951 up to 1956, the latest year for which results were then available. Results are now available from the three Observatories of the Meteorological Office up to the end of 1958* and annual mean values are given in Table 1, together with values from two Portuguese Observatories†, the only similar data available to us.

The quantities tabulated are means for ' quiet days ' selected according to the usual conventions, the annual means being found as the mean of the twelve monthly means for each year, irrespective of the number of quiet days in each month. The values for Kew are observations of potential gradient and air-earth current made each fine afternoon at about 1430 GMT, using the Wilson test-plate method in an underground laboratory where the lead roof is level with the surrounding grass lawn; the positive-ion conductivity at ground level has been deduced from the ratio of the current and potential gradient observations. Values of potential gradient for the other stations are derived from recording electrometers connected to suitable ' collectors.' The electrometer readings are converted to potential gradient values by means of frequent standardizing observations made at a height of one metre over an open, level site.

It will be seen that very marked changes in atmospheric electricity have occurred at all five stations in the last few years. At Eskdalemuir, Lisbon and Oporto the potential gradient has been roughly halved and at Lerwick it appears to have fallen by some 25 per cent. At Kew there has been no significant change in potential gradient but the air-earth current and the positive-ion conductivity have increased by a factor of almost two. It is clear from Table 1 that these changes in the last few years are quite outside the range of variation experienced at the British stations before 1952. The possibility that the apparent changes are due to faulty measurements has been carefully examined and independent checks on the measurements at Kew and Eskdalemuir have been made, without disclosing any significant errors in the data of Table 1 (though a few minor corrections to the data given by Pierce will be noticed). Fuller details of the methods of measurement are given in the *Observatories Year Book* (1938) and in a paper by Scrase (1934).

2. Possible causes for the observed changes

The potential gradient near the ground in fine weather F depends on the potential of the conducting layer high in the atmosphere V, the resistance of a column of air of unit

* In course of publication, *Observatories Year Book* (H.M.S.O.).

† Taken from *Boletim Geoelectrico* of the Portuguese Servico Meteorologico Nacional.

cross-section extending from this layer to the ground R and the specific resistance, or conductivity, λ, of the air near the ground, being given by $F = \frac{V}{R\lambda}$. Changes in F could be caused by changes in V or R but these would involve very significant changes throughout the whole atmosphere and it seems much more likely that the observed changes in F are due to changes in λ, changes which may be confined to the lowest few metres of the atmosphere, where the electrical measurements have been made. The conductivity of the air is roughly proportional to the number of small ions present and this, in turn, depends on a balance between the production of ions and their disappearance through recombination or attachment to suspended particles, or diffusion.

A general decrease in potential gradient near the ground can thus be attributed either to a decrease in the number of suspended particles in the air or to an increase in its rate of ionization, such as might be caused by artificial radioactive material in the air or on the ground.

A record of the amount of smoke suspended in the air has been maintained at Kew since 1921 and at Eskdalemuir and Lerwick since 1949. Annual mean values are included in Table 1. It will be seen that the smoke pollution at Eskdalemuir and Lerwick is very small and that neither at these stations nor at Kew is there any indication of a change in pollution which could explain the electrical changes.

TABLE 1. ANNUAL MEAN VALUES FOR ATMOSPHERIC ELECTRICITY AND SMOKE POLLUTION

F = potential gradient, v m^{-1} — S = smoke pollution, mg m^{-3}
i = air-earth current, 10^{-16} a cm^{-2} — λ = positive-ion conductivity, 10^{-18} Ω^{-1} cm^{-1}
– = values missing through instrumental defects

	Lerwick		Eskdalemuir		Kew				Oporto	Lisbon
	F	S	F	S	F	S	i	λ	F	F
1938	183		214		302	·09	1·08	44		
1939	174		217		332	·09	1·11	43		
1940	168		231		297	·09	0·96	44		
1941	157		202		263	·14	0·96	47		
1942	170		217		338	·14	1·12	41		
1943	188		213		278	·14	0·73	33		
1944	166		225		299	·12	0·87	34		
1945	195		224		321	·11	1·05	40		
1946	184		223		357	·08	1·22	42		
1947	173		210		292	·07	0·93	39		
1948	180		219		307	·11	1·01	47		
1949	173		193		305	·14	–	–		
1950	185	·003	201	·010	328	·15	–	–		
1951	200	·004	224	·009	362	·20	–	–		
1952	154	·002	188	·007	363	·18	–	–		
1953	190	·003	182	·008	359	·14	–	–		
1954	200	·002	153	·007	348	·08	–	–	142	
1955	172	·003	143	·013	–	·09	–	–	128	105
1956	141	·005	133	·014	293	·09	1·50	57	136	98
1957	125	·006	92	·011	347	·10	1·81	64	102	76
1958	3	·002	90	·010	319	·11	1·97	74	85	60

It is well known that much radioactive material has been released into the air by nuclear explosions in the last few years, and many measurements of its concentration in the air and on the ground have been published. Israel (1958) has shown convincingly that the measured mean concentration of about 2×10^{-18} curies cm^{-3} of fission products in the air near the ground would increase the conductivity there by only about 3 parts in 10^4. On the other hand, the accumulation of fission products on the ground may have

much larger effects. Israel concluded that the quantity brought down by quite small amounts of precipitation would profoundly affect the conductivity near the ground were it not for the fact that the fission products were washed into the soil by the rain. Pierce (1959) estimated that this penetration of fission products into the soil would reduce the ionization they caused in the air by a factor of 15. Using this factor, he calculated that the quantity of the radioisotope Sr90 known to have been deposited in the region of Eskdalemuir up to the end of 1957 would cause an extra ionization of about $0{\cdot}2I$ (I is used to denote ion-pairs cm^{-3} sec^{-1}) in the air near the ground; this is about 50 times too small to explain the halving of the potential gradient. However, recent reports (e.g. Stewart *et al.* 1957, 1959; Peirson and Salmon, 1959) suggest that the total fission product radioactivity on the ground during most of the period 1954-58 was up to 100 times greater than that due to Sr90 alone, through the continually renewed presence of substantial amounts of relatively short-lived fission products. On this basis it seems very likely that the changes noted in Table 1 have been caused by the fission products from explosions, but it is unsatisfactory that the estimates of extra ionization have to depend on the large and uncertain factor that must be allowed for the penetration of the radioactivity into the soil. It was therefore decided to make direct measurements of the extra ionization produced in the air by surface radioactivity.

3. Measurements of ionization by β-rays

The natural ionizing agents near the ground are cosmic rays (producing about $2I$), α-rays from radon and thoron in the air ($5I$), and β- and γ-rays from radioactive matter in the ground ($2I$ each). Fission products deposited on the ground will emit β- and γ-rays in roughly equal quantities, but near the ground surface the ionization due to the β-rays will be of the order of 100 times that due to the γ-rays because of the shorter range and greater specific ionization of the β-rays. This difference does not appear with the natural β- and γ-rays because the radioactive material is distributed in depth and β-rays from the lower layers are absorbed much more than γ-rays.

The extra ionization we wish to measure is thus almost entirely due to β-rays; even if the γ-ray intensity is only a fairly small fraction of the natural background of γ- and cosmic-rays, as implied by many published statements, the β-ray intensity may still be enough to double the natural rate of ionization near the ground.

Measurements have been made using ionization chambers very similar to that described by Hess and O'Donnell (1951). They consisted of shallow rectangular boxes of aluminium $28 \times 28 \times 7{\cdot}7$ cm^3, the top and sides being 0·32 cm thick. The bottoms were covered with aluminium foil 0·0025 cm thick (7·0 mg cm^{-2}). The central electrode in each chamber was an aluminium rod, 0·32 cm in diameter, bent into a square of side 15 cm and emerging through 'fluon' insulation surrounded by a guard-ring insulated from the side of the chamber. The chambers were left full of ordinary air and no special precautions were taken to avoid air leaks. In order to avoid difficulties with residual ionization in the chamber and fluctuations in battery voltage, two chambers were used, connected differentially. They were arranged 15 cm apart in the same horizontal plane; the central electrodes were connected together and to a Lindemann electrometer mounted just above the chambers. In use, one chamber was made about 60 v positive and the other 60 v negative with respect to earth, the voltages (which were sufficient to give saturation current) being obtained from a single battery and adjusted so that the charges induced on the central electrodes were equal and opposite and the needle of the electrometer remained at earth potential.

Measurements of the β-rays reaching the chambers from below were made by placing an aluminium sheet 0·32 cm thick beneath one or other chamber. This left one chamber exposed to β-rays while the other was almost completely screened from them, though the two chambers were equally exposed to γ-rays and cosmic rays. The difference in ionization currents, measured by observing the drift of the electrometer needle over

periods of five or ten minutes, was a measure of the ionization due to β-rays in one chamber. By taking readings with the chambers exposed alternately, the effects of steady leaks and zero drifts in the electrometer were eliminated; variations in the zero drift set the limit to the accuracy with which measurements could be made – the current was usually uncertain by about 7×10^{-16} a, corresponding to an ionization of about 1 *I*. An alternative method of avoiding leakage, by continually adjusting the potential of the ionization chambers so as to maintain the central electrode at earth potential, gave less satisfactory results.

The effects of β-ray absorption in the thin foils and γ-ray absorption in the plates used to cut off β-rays were measured by experiments using double thickness of foils or plates. The effects were small (8 per cent) and in typical conditions were almost equal and opposite, so no corrections were applied to allow for them.

The capacity *C*, of the electrometer and electrode system was measured with the help of a known high resistance and found to be 37×10^{-12} farads. The electrometer was calibrated for each set of readings, its sensitivity, *S*, being about 40 division per volt; readings were taken to 0·1 division. If the electrometer deflection was θ divisions in *t* seconds the rate of production of ions was calculated as $\theta C/StVe$ ion pairs cm^{-3} sec^{-1}, where *e* is the electronic charge and *V* the effective volume of one chamber. The actual volume of each chamber was 6,000 cm^3 and the effective volume, for an infinite plane source was estimated to be 4,500 cm^3 (with an uncertainty of about 20 per cent).

4. Results of measurements

The results of measurements made at three British Observatories in October and November 1959 are given in Table 2. At each Observatory measurements were made over short grass at several sites selected at random in the general neighbourhood of the site of the standard electrical observations. At Eskdalemuir and Lerwick measurements were also made over the rough moorland which surrounds these Observatories. (At Lerwick, the standard electrical observations are made over a narrow strip of levelled ground (20 m × 1 m) on the moor outside the Observatory enclosure). In addition, at each station, measurements were made over shallow, newly-dug pits (about 120 cm × 60 cm × 10 cm deep), referred to in the Table as ' fresh bare earth '; it was hoped that these measurements would correspond roughly to the natural β-ray activity, because little of the fission products would penetrate more than 10 cm into the ground.

These measurements were all made as close to the ground as practicable – at a height of about 5 cm for short grass and about 12 cm for moorland – the height being measured from the bottom of a wooden block resting on the ground to the bottom of the ionization chambers.

At Kew, measurements were also made at heights of 60 and 170 cm over short grass, over the lead roof of the underground electrical laboratory and over the 2 m-wide strip of asphalt that surrounds this roof. This asphalt was newly laid at the time of the observations and it is hoped that by re-measuring the ionization over it from time to time some indication of the rate of deposition of fission products may be obtained. The old asphalt, laid 25 years earlier and in a bad state of repair, was taken up in September 1959 and a slab of it was brought indoors and placed under the ionization chambers, firstly with its original top surface uppermost, and then inverted so that its old under-side was towards the chambers. This slab has since been kept indoors with the idea of following any decay of its surface radioactivity by repeating the measurements from time to time.

The measurements are expressed in Table 2 as the number of ion pairs produced by β-rays in one cubic centimetre of air in one second (*I*). Each value was derived from the mean of several (usually 5 to 10) electrometer readings with the chambers exposed alternately, as explained above. The scatter of these readings varied, depending on the steadiness of the electrometer drift, but the standard error of the mean values in the table is 1*I* or less, except for the moorland reading at Lerwick, where it is about 3*I*.

TABLE 2. IONIZATION BY β-RAYS (ION-PAIRS cm^{-3} sec^{-1})

Date 1959	Site	Description	Height cm	Ionization I
		Kew Observatory (51° 28′N, 0° 19′W)		
2 October	K1	Grass lawn, south of electrical laboratory	5	11·4
6 October	K2	New asphalt surround to electrical laboratory	5	1·2
	K3	Lead roof of electrical laboratory	5	11·6
	K4	Grass lawn, north of electrical laboratory	5	14·5
	K4	Grass lawn, north of electrical laboratory	60	10·8
7 October	K5	Grass lawn, north of electrical laboratory	170	3·7
	K4	Grass lawn, north of electrical laboratory	5	13·3
	K6	Fresh bare earth close to K4	5	2·8
15 October	K4	Grass lawn	5	13·7
29 October	K4	Grass lawn, after heavy rain	5	7·3
	K1	Grass lawn, after heavy rain	5	9·1
5 November	K3	Lead roof	5	10·9
	K4	Grass lawn	5	6·9
	K2	New asphalt	5	1·7
30 September	–	Old asphalt slab, indoors	5	11·6
5 October	–	Old asphalt slab, indoors	5	11·2
	–	Old asphalt slab, indoors inverted	5	1·2
3 November	–	Old asphalt slab, indoors	5	9·0
		Lerwick Observatory (60° 08′N, 1° 11′W)		
22 October	L1	Grass lawn, meteorological enclosure	5	8·8
23 October	L2	Grass lawn, meteorological enclosure	5	9·3
	L3	Fresh bare earth, close to L2	5	− 0·6
24 October	L4	Moorland near site of absolute electrical observations	12	7·4
		Eskdalemuir Observatory (55° 19′N, 3° 12′W)		
25 October	E1	Grass lawn, northwest of electrical pit	5	18·8
	E2	Fresh bare earth, close to E1	5	3·6
	E3	Grass lawn, close to Observatory building	5	9·4
	E4	Grass lawn, close to electrical pit	5	17·1
26 October	E5	Grass lawn, between E1 and E4	5	11·6
	E1	Grass lawn	5	13·0
	E6	Moorland, 50 yards north of boundary wall	12	10·4

5. DISCUSSION

As might be expected, Table 2 shows that there are fairly wide variations in ionization from one site to another at each station, even over apparently similar surfaces. There are also changes of up to 50 per cent when repeated measurements are made at the same site (e.g., sites K4 and E1). These changes seem to be correlated with the wetness of the ground. Up to 20 October the grass at Kew was very dry and the readings at site K4 on 6, 7 and 15 October show little change; by 29 October the grass was thoroughly wet and the readings at K1 and K4 (though not those on the lead roof, which was dry when observations were made) were considerably smaller than the previous ones. The reduction in ionization could be explained by the absorption of β-rays in a layer of water about 1 mm thick on the ground surface, and it seems quite likely that the effective thickness of the water in the upper layers of the wet soil and on the wet grass was of this amount.

The ground at Lerwick and Eskdalemuir was very wet during the measurements there (rain was actually falling while some of the measurements were made) and the values of ionization obtained are, therefore, probably rather too low to represent annual mean conditions. On the other hand, most of the values at Kew, obtained after a long dry

spell, are probably above the annual average. The effect of water on the ground is so great that a long series of regular measurements would be needed to get a reliable mean value.

It is clear, however, that the general level of β-ionization at all three stations is considerably greater than the natural background measured over fresh earth, and we can take figures of $10I$, $15I$ and $8I$ as representing roughly the extra ionization produced at a height of 10 cm by β-rays from material deposited on the ground surface at Lerwick, Eskdalemuir and Kew respectively, when the ground has average wetness. The measurements at Kew show that this ionization falls off with increasing height, the value at one metre being about half that near the ground.

It is clear that this extra ionization will have a considerable effect on measurements of atmospheric electricity, but it is difficult to estimate the effect precisely. The concentration of small ions, and hence the conductivity, near the ground will be roughly proportional to the rate of production of ions, but since the air is normally mixed by turbulence it is a mean rate of production throughout a height of order $(K\tau)^{\frac{1}{2}}$ that controls the equilibrium concentration of ions, where K is the eddy diffusivity of the air and τ the mean life of a small ion. Taking typical values of 10^3 cm^2 sec^{-1} for K and 40 sec for τ, the height of the layer to be considered is of order 2 m. For such a layer the mean extra ionization by β-rays will be about one-half the values given above for the ionization at 10 cm from the ground. This is to be compared with a natural rate of ionization usually taken to be about $10I$ (Chalmers 1957), though considerable local variations are to be expected. No estimates of natural ionization at Lerwick or Eskdalemuir are available, but for Kew, Scrase (1935) found a value of only $4{\cdot}5I$ (and we may note that some measurements by Hogg (1939) suggest that only a small fraction of this was due to β-rays). It thus seems that the extra ionization observed in the autumn of 1959 would be expected to increase the conductivity of the air in the lowest few metres by a factor of about two, but in view of the uncertainties about the natural level of ionization, the effects of surface water on the β-rays and the conditions governing the equilibrium number of ions, this factor must be regarded as a very rough estimate. The ionization measurements suggest that it should be larger at Lerwick than at Kew and still larger at Eskdalemuir, but this effect could easily be swamped by other differences between the three places, such as differences in the natural level of ionization, the degree of turbulence in the air or the nearness of the sea.

An increase in the conductivity of the air near the ground would be expected to produce a proportional decrease in the potential gradient near the ground, as indicated in Section 2. This has occurred at Lerwick, Eskdalemuir, Lisbon and Oporto and there seems little doubt that the observed changes in potential gradient can be attributed to the effects of radioactive material deposited on the ground surface. At Kew an increase of conductivity near the ground has been observed directly, and can be attributed to the same cause. It is not clear why at Kew this increase in conductivity is accompanied not by a fall in potential gradient but by an increase in air-earth current. Two speculative suggestions can be put forward. The conductivity at Kew has always been abnormally low, presumably because of heavy smoke pollution. It is likely that considerable variations of space charge exist in the air above Kew and that a large fraction of the air-earth current is carried by convection of space charge rather than by ionic conduction. The effect of the recent increase of conductivity near the ground would be to increase the gradient of space charge there and so to increase the convection currents. In this way the local increase in conductivity near the ground might simulate the effects of an increase through a much greater depth, producing the increased air-earth current and approximately constant potential gradient that have been observed. Attempts to treat this mechanism quantitatively suggest that it cannot be a complete explanation, though it may be a partial one. The second suggestion is that recent increases in motor traffic on the roads near Kew may have increased the generation of positive space charge in the neighbourhood. This, by itself, would be expected to increase the potential gradient and the air-earth current, but in combination with the observed increase in conductivity near the ground it could result in constant potential gradient and increased current.

6. Conclusions

The observations collected in Table 1 show that large changes in the electrical state of the atmosphere near the ground have taken place over a wide area since 1952. The observations presented in Table 2 show that at three places in Britain in the autumn of 1959 extra ionization of the air by β-rays from material concentrated in the top few centimetres of the ground was roughly equal to the natural background of ionization and was therefore quite sufficient to explain the observed electrical changes. The β-rays almost certainly come from fission products deposited on the ground as a result of nuclear explosions; their intensity is about what would be expected from measurements of the amounts of fission products deposited. Most of the β-rays come from relatively short-lived isotopes, and the extra ionization, and hence the electrical measurements, may be expected to show variations corresponding to variations in the frequency and magnitude of nuclear explosions, with a time lag of the order of one year.

Although these changes in ionization and electrical properties are so large near the ground, there is no evidence that they extend to heights greater than a few metres (except, perhaps, in the rather unusual conditions at Kew) and they would not necessarily be expected to have any appreciable effect on large-scale electrical phenomena.

Acknowledgments

I am indebted to the Director-General of the Meteorological Office for permission to publish this paper, to the Director of the Warren Springs Laboratory, D.S.I.R. for providing some of the smoke-pollution data and to my colleagues at the Observatories for much helpful discussion and for assistance with the ionization measurements.

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551.574.1 : 551.594.21

Ice-crystal contact electrification

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(Manuscript received 22 February 1960)

SUMMARY

An experimental investigation of electrification accompanying the momentary single contact of vapour-grown ice crystals at the same or at different temperatures suggests than any such effect is much smaller than might be expected from Reynolds's report on charging observed with riming spheres and vapour-grown ice crystals.

1. INTRODUCTION

A preliminary investigation has been made into any electrification which may accompany the momentary contact of two samples of ice, at the same or at different temperatures, and grown from the vapour phase as crystals.

Simpson and Scrase (1937) suggested that the main positive dipole of a thundercloud was due to ice-impact electrification. Reynolds (1954, p. 77) describes experiments in which warmer ice coming momentarily into contact with colder ice acquires a negative charge, and uses this result to explain the charging of a pair of whirling ice-covered spheres as they collide with ice crystals and supercooled water droplets together. In a thundercloud the main positive dipole appears to be due to an extensive accumulation of positive charge at temperatures around − 20°C, and, lower down, an accumulation of negative charge in the neighbourhood of the − 5°C isotherm. There seems to be a close connexion between the appearance of this dipole and the occurrence of precipitation, often as graupel. At these temperatures it is possible that supercooled water, or ice, or both together, will play a part in the main dipole electrification process. A lower *positive* centre of charge often found near the cloud base may be related to the liquid phase rather than to the solid.

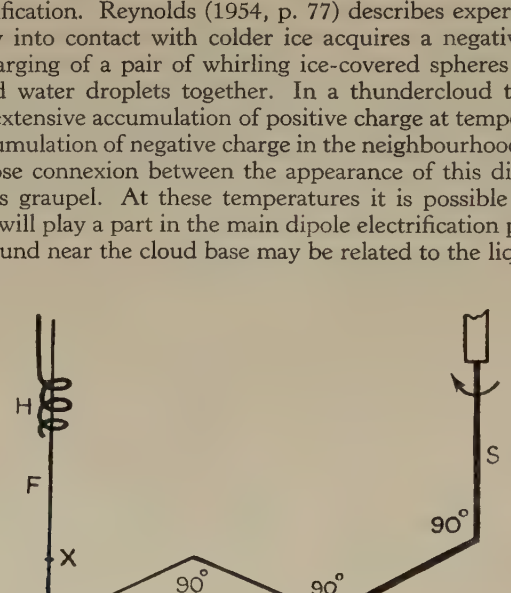

Figure 1. The moving parts for effecting the contact.

H = Faraday cylinder helix.
F = Fibre.
M = Movable horizontal fine wire.
S = Supporting member which may rotate so that *M* touches *F* at *X*.

2. EXPERIMENTAL PROCEDURE

In the present investigation crystals were grown, from the vapour, on a vertical insulating fibre *F*, made by drawing out a drop of *Durofix* adhesive, and kept taut by a tiny weight *W*. This was suspended in a diffusion chamber 10 cm deep and having a cross-section 30 cm square. The temperature, measured by means of fine horizontal thermocouple wires, increased fairly steadily from bottom to top, typical values at 0, 2, 4 and 6 cm being − 28, − 16, − 10 and − 5°C. The fibre *F* could be raised quickly so that any point in it was inside a Faraday cylinder in the form of a very open helix *H*, which was 20 mm long and 8 mm in diameter, and consisted of three turns of 40 s.w.g. copper wire. The helix was connected to the highly insulated terminal

of a vibrating-reed electrometer adapted for measuring single charges. The earthed metal parts of the diffusion chamber gave adequate electrostatic screening. Thus any charge at a point on the fibre could be measured. A careful preliminary study showed that with this particular helix it was possible to make reliable measurements of charge, the value indicated being 59 per cent of the true value. The limit of sensitivity of the apparatus was approximately $0{\cdot}02 \times 10^{-12}$ C. All wires in the centre of the diffusion chamber were fine, and horizontal if possible, to minimize heat flow along them. There was also a horizontal fine wire M which could be moved in a horizontal plane by causing a stouter supporting member S to rotate about a vertical axis. Crystals growing on this wire could thus be brought momentarily into contact with those on the fibre – though there was often sticking and entanglement so that it was difficult to separate the crystals after contact. The procedure for making an observation was as follows. First, M and W were each set to a suitable level. When the crystal growth on them was considered to be sufficient some point X in the ice growth on F was chosen for contact with ice on M. The fibre was quickly raised or lowered until X and M were level. M was brought towards X, the ice-ice contact quickly made and broken, M was withdrawn, and the fibre quickly raised until X was in the middle of the helix and any charge was measured. The fibre insulation was high, the leakage time constant being about 30 min. There was no perceptible charging of the fibre owing to moving it up and down. Observations at a point on the fibre could always be, and were, checked by noting one indication when the point entered the helix, and another when it was afterwards lowered out of the helix. Spurious charges, due to accidental, or deliberate, contact with parts of the apparatus were always available for checking that the charge-measuring equipment remained sensitive and that insulation was good. The temperature of the two ices was inferred from the actual level and the temperature-height dependence. The area of the surfaces in contact was estimated to lie between 0·2 and 2 mm^2, and the duration of contact usually between 0·2 and 0·5 sec, but sometimes as much as 1 sec.

The diffusion chamber was cooled by circulating paraffin oil, itself cooled by solid carbon dioxide to about -40°C, through the base. Convection was largely avoided by building inside the chamber an inner compartment having perspex sides, and there were perspex windows, with double air-spacing, for illuminating and viewing the crystals. The crystals on the fibre, although several millimetres long, did not show clearly the temperature-determined habit described by Hallett and Mason (1958), but Dr. B. J. Mason, on a visit to this laboratory, has pointed out that adequate supersaturation was probably prevented by the influx of nuclei through the air space which was considered essential to maintain the insulation of the helix.

3. Conclusions

It was found that whether the contact between the crystals was made gently or violently, and whether or not there was any initial temperature difference, no contact charging was indicated. The maximum temperature difference was estimated to be 14°C. Occasionally there appeared persistent charges of the order of $0{\cdot}1 \times 10^{-12}$ C, but these usually occurred after an accidental contact with the apparatus, and were assumed to be spurious, as previously mentioned. It may be concluded that any charges produced had magnitudes below the sensitivity limit of the apparatus.

It is of interest to consider this result, for *single* contacts, with that of Reynolds (1954, p. 85) for the numerous collisions made by his whirling ice-covered sphere. He reports an approximate average value per collision some 10 times the sensitivity limit of the apparatus described above. Although temperature difference figures are not given, an estimate of 2°C is quoted for comparable natural conditions. These temperatures differences are much smaller than in the work reported here, yet the average charge per collision was appreciably greater. It seems unlikely that the charging observed by Reynolds was due to temperature difference alone. In his report Reynolds also deals with charging associated with contaminants present in the ice in great dilution. Such effects have not yet been investigated in this laboratory.

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Simpson, G. C. and Scrase, F. J. 1937 'The distribution of electricity in thunderclouds,' *Proc. Roy. Soc., A*, London, **161**, p. 309.

551.508.77 : 551.578

A method of obtaining average bearing and incidence of rainfall

By C. W. ROSE and H. G. FARBROTHER
Makerere University College, Uganda
Empire Cotton Growing Corporation, Namulonge, Uganda

(Communicated by H. L. Penman; manuscript received 1 February 1960)

Summary

Three tilted raingauges are symmetrically disposed round a vertical axis, with their collecting openings facing outward at an angle of 45° to the horizontal. From the ratios of the catches the mean direction of travel of a rainstorm can be estimated, and also the ratio of horizontal to vertical velocity of raindrops, provided it is less than unity. At Namulonge, Uganda (17 mi N of Kampala), between September 1954 and May 1956, the most common direction of rainfall was from the south, in the sector 150° to 210°, and the velocity ratio exceeded tan 10° for 90 per cent of the total rain.

1. Introduction

It is known (Faulwether 1917) that bacterial blight on cotton plants can be spread by raindrop splash, and as part of a disease study at the research station of the Empire Cotton Growing Corporation in Uganda, an instrument was built to measure the two components of direction

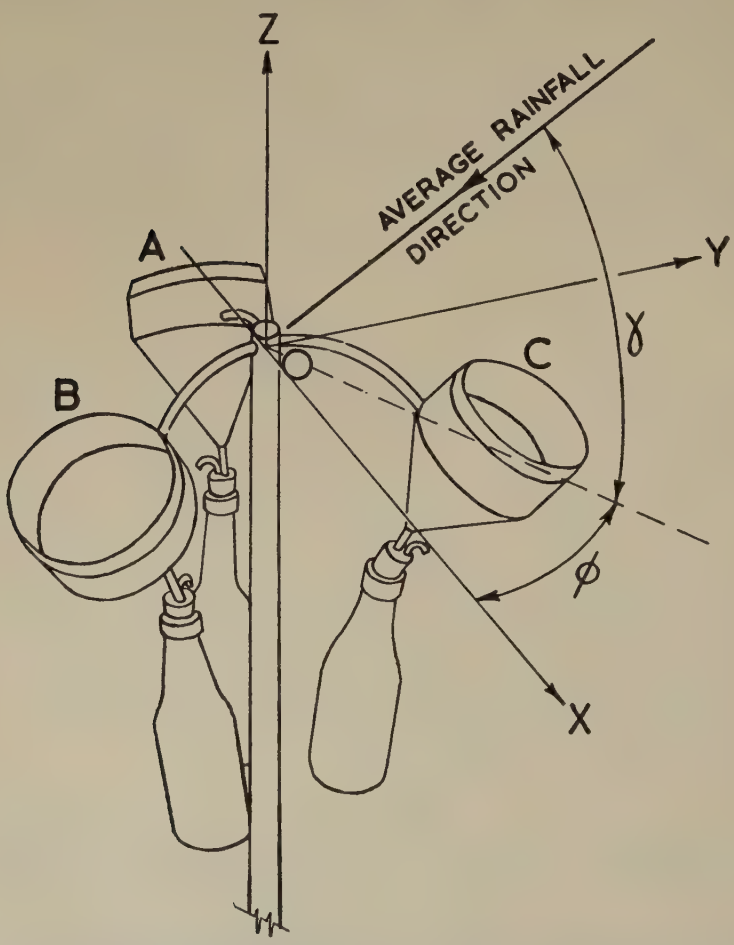

Figure 1. The three-raingauge instrument devised by H. G. Farbrother. The plane defined by the Cartesian axes OX and OZ contains the axis of rain-collector A.

needed to specify the angle of impact of raindrops on the plant. The instrument is thought to be simpler in construction than others designed for similar purposes (e.g. Fourcade 1942, Lacy 1951), and the required information is easily obtained from prepared charts.

2. The instrument

Three standard rain-collectors are mounted symmetrically at the top of a vertical post on curved brackets so that the plane containing the rim of a collector makes an angle of 45° with the horizontal. Each collector has its own collecting bottle.

3. Theory of instrument

Let Q_a, Q_b, Q_c be the quantities of rain caught in the three collectors, in ascending order of magnitude. (Equal catches cause no difficulty).

Let ϕ be the average bearing of the rainfall, measured from the line of the axis of collector A (least catch), as shown in Fig. 1 and in the inset of Fig. 2 in which A, B, and C represent the three raingauges in plan.

Let γ be the average angle of incidence of the rainfall measured from the horizontal, so that $\tan \gamma =$ vertical velocity/horizontal velocity.

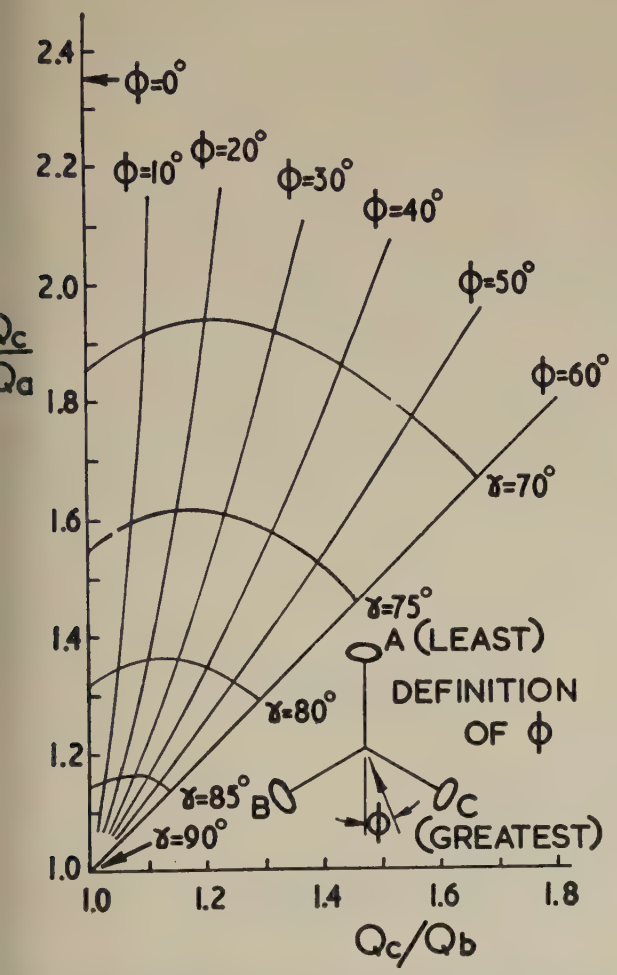

Figure 2. Graph for obtaining γ and ϕ (see text), for larger values of γ. Q_c must be the greatest and Q_a the least rainfall catch.

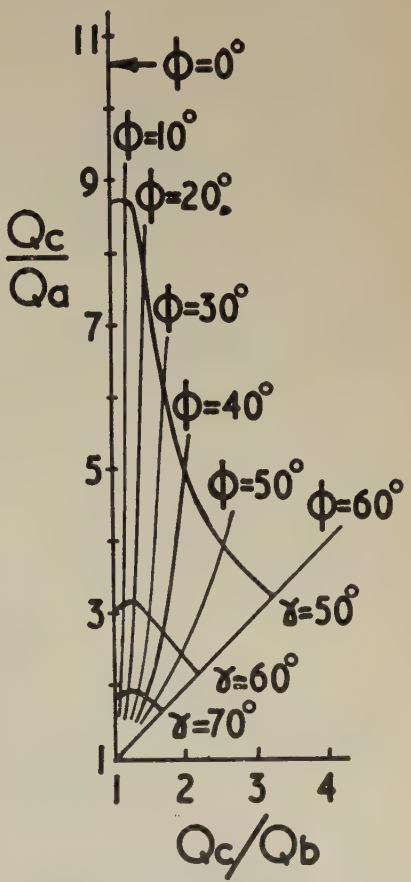

Figure 3. Same as Figure 2, but for lower values of γ.

The total quantity of rainfall caught in any collector is proportional to the component of its entry area as viewed from the (vector) average direction of rainfall (ϕ, γ). The normal to the entry area of any rain-collector can be defined by the modified spherical polar co-ordinates (α, β) with respect to the same Cartesian axes OX, OY, OZ, (Fig. 1) used to define ϕ and γ. From Fig. 1, for collector C, $\alpha_c = 60°$ and $\beta_c = 45°$; for A, $\alpha_a = 180°$ and $\beta_a = 45°$; for B, $\alpha_b = 300°$ and $\beta_b = 45°$. From solid geometry theory the angle θ between the two lines whose co-ordinates are (ϕ, γ) and (α, β) is given by :

$$\cos\theta = \cos\alpha\cos\beta\cos\phi\cos\gamma + \\ + \sin\alpha\cos\beta\sin\phi\cos\gamma + \sin\beta\sin\gamma \quad . \quad . \quad . \quad (1)$$

Let θ_a, θ_b, θ_c represent the angles between the direction of average rainfall and the normals to collectors A, B, and C respectively. Substitution into Eq. (1) of the values of the co-ordinates of the collector normals (α_a, β_a, etc. given above) yields :

$$\cos\theta_a = -0{\cdot}706\cos\phi\cos\gamma + 0{\cdot}707\sin\gamma$$

$$\cos\theta_b = 0{\cdot}354\cos\phi\cos\gamma - 0{\cdot}6125\sin\phi\cos\gamma \\ + 0{\cdot}707\sin\gamma$$

$$\cos\theta_c = 0{\cdot}354\cos\phi\cos\gamma + 0{\cdot}6125\sin\phi\cos\gamma \\ + 0{\cdot}707\sin\gamma$$

Using these expressions $\cos\theta_a$, $\cos\theta_b$ and $\cos\theta_c$ were computed for a suitably spaced selection of all allowable combinations of ϕ and γ, i.e. $0° \leqslant \phi \leqslant 60°$, $45° < \gamma \leqslant 90°$.

Let A_a = component of entry area of collector A as viewed from the average rainfall direction. Then $A_a = N\cos\theta_a$, where N = normal entry area of each collector. There are corresponding expressions for collectors B and C.

Hence $Q_a : Q_b : Q_c = A_a : A_b : A_c = \cos\theta_a : \cos\theta_b : \cos\theta_c$.

Each intersection in Fig. 2 and 3 is the result of computation from this theory. If either graph is entered with the ratio of the greatest to the least rainfall catch as ordinate, and the greatest to the second greatest catch as abscissa, a point on the graph is defined. The values of ϕ and γ can then be read off by interpolation. A value of ϕ can be interpreted in terms of geographical bearing if the orientation of the three rain-collectors is known.

The shape which develops in lines of constant γ as γ decreases, and which is pronounced in the curve for $\gamma = 50°$ shown in Fig. 3, is caused by the raingauges being inclined at an angle of 45° to the horizontal, so that one collector will not be receiving rainfall if $\gamma < 45°$ for $\phi = 0°$. For such a situation the ordinate becomes infinite, and the analysis is no longer valid. If a low average incidence value is indicated (say 50°), it is quite possible that during some periods of rainfall γ is less than 45°, thus introducing an error; but such errors will decrease rapidly as average incidence increases above 45°. In addition, there will almost certainly be errors because of rainfall splashing out of the collectors.

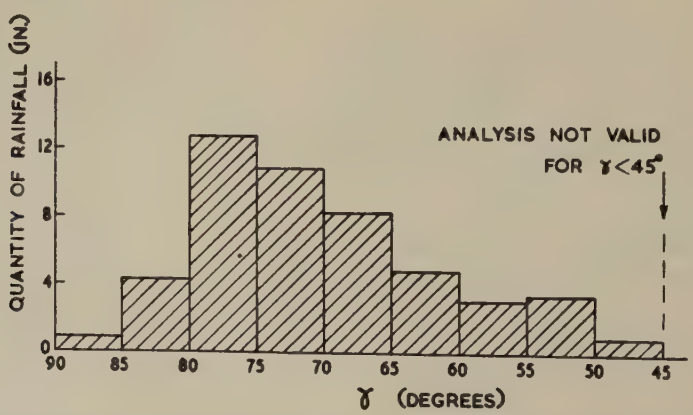

Figure 4. Histogram showing quantity of rain received on a horizontal surface (at the Empire Cotton Growing research station, Namulonge, Uganda, over the period 2 Sept. 1954 to 4 May 1956) plotted against average angle of rainfall incidence (γ), measured from the horizontal.

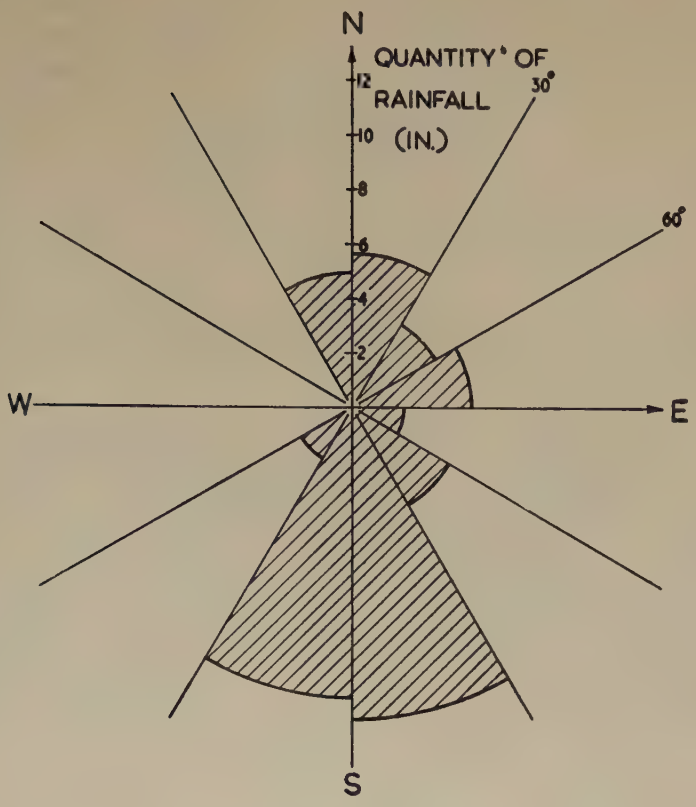

Figure 5. The quantity of rainfall received on a horizontal surface (plotted radially) for 30° sectors of geographical bearing. The situation and period is the same as in Fig. 4.

4. Experimental results

In the results summarized in Fig. 4 (for γ) and Fig. 5 (for ϕ), rainfalls of less than 0·1 in. have not been included, and for various reasons a few occasions of higher rainfall have been omitted.

Fig. 4 shows that 90 per cent of the rainfall in the 20-month period was deflected by wind through an angle greater than 10° from the vertical (i.e., $\gamma < 80°$). It would appear that the contribution was very small from rainfalls for which the average angle of incidence was less than 45° to the horizontal. Hence an angle of tilt (β) of 45° on the raingauges was suitable, though another value for β may be better in a region with rainfall of a very different character. If a value for β different from 45° was adopted, the general analysis of Section 3 could be used to compute the required graphs similar to Figs. 2 and 3.

From Fig. 5, the most common direction of rainfall was in the sector 150° to 210°, i.e., generally from the direction of Lake Victoria to the south.

Acknowledgment

The assistance of Dr. H. L. Penman in preparing this article is gratefully acknowledged.

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Faulwether, R. C. 1917 *J. Agric. Res.*, **10**, p. 639.
Fourcade, H. G. 1942 *Roy. Soc., S. Africa*, **29**, p. 235.
Lacy, R. E. 1951 *Quart. J. R. Met. Soc.*, **77**, p. 283.

551.573 : 551.577.3 : 551.579.

Differing water budgets in two adjacent reservoirs

By PAMELA EDWARDS
National Institute of Oceanography

(Manuscript received 29 February 1960. Communicated by H. Charnock)

SUMMARY

Before making experiments with surface films intended to reduce evaporation it was necessary to compar the variations of water level in two similar adjacent reservoirs. Although they were in relatively flat countr significant differences in their water budgets appeared; they are described in this note.

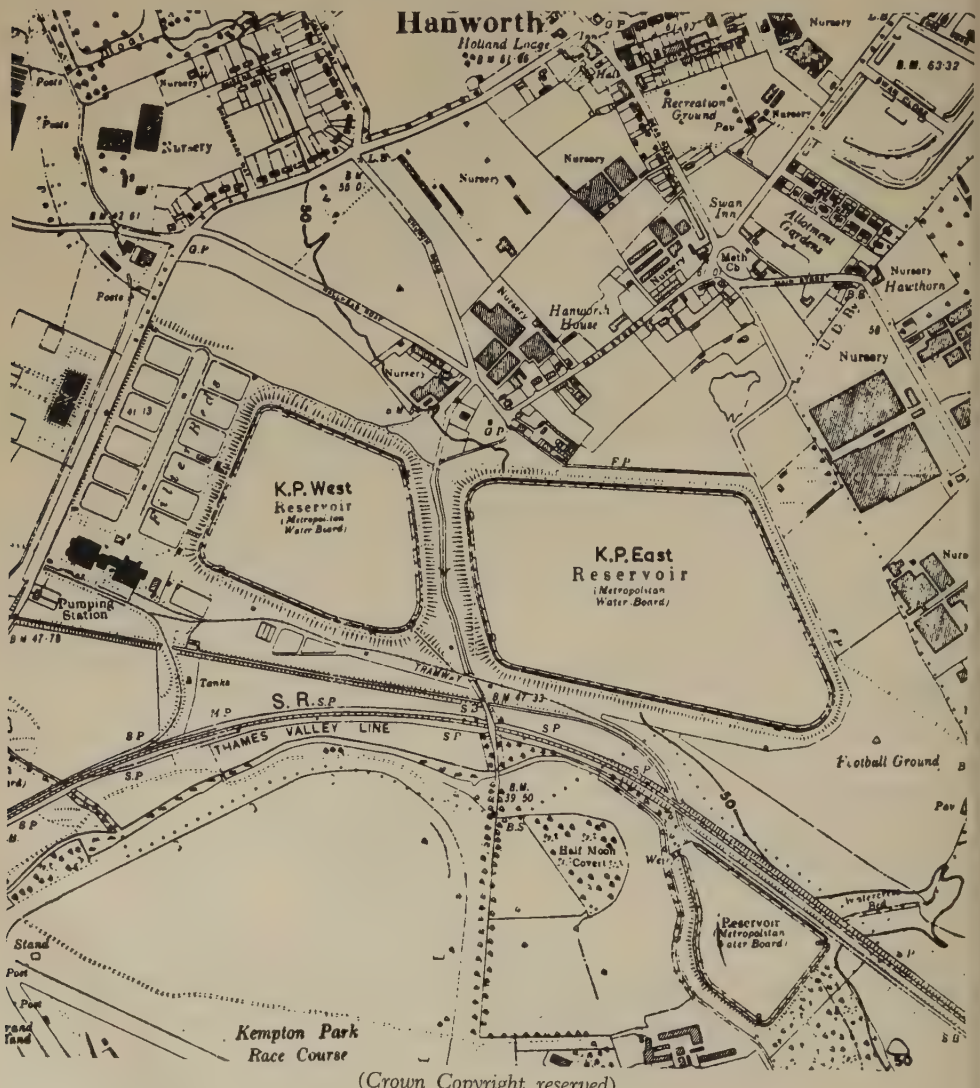

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Figure 1. Map showing situation of Kempton Park East and Kempton Park West Reservoirs. The East Reservoir is 40 acres in area and the West 20 acres. The longest side of the East Reservoir is about one-third of a mile.

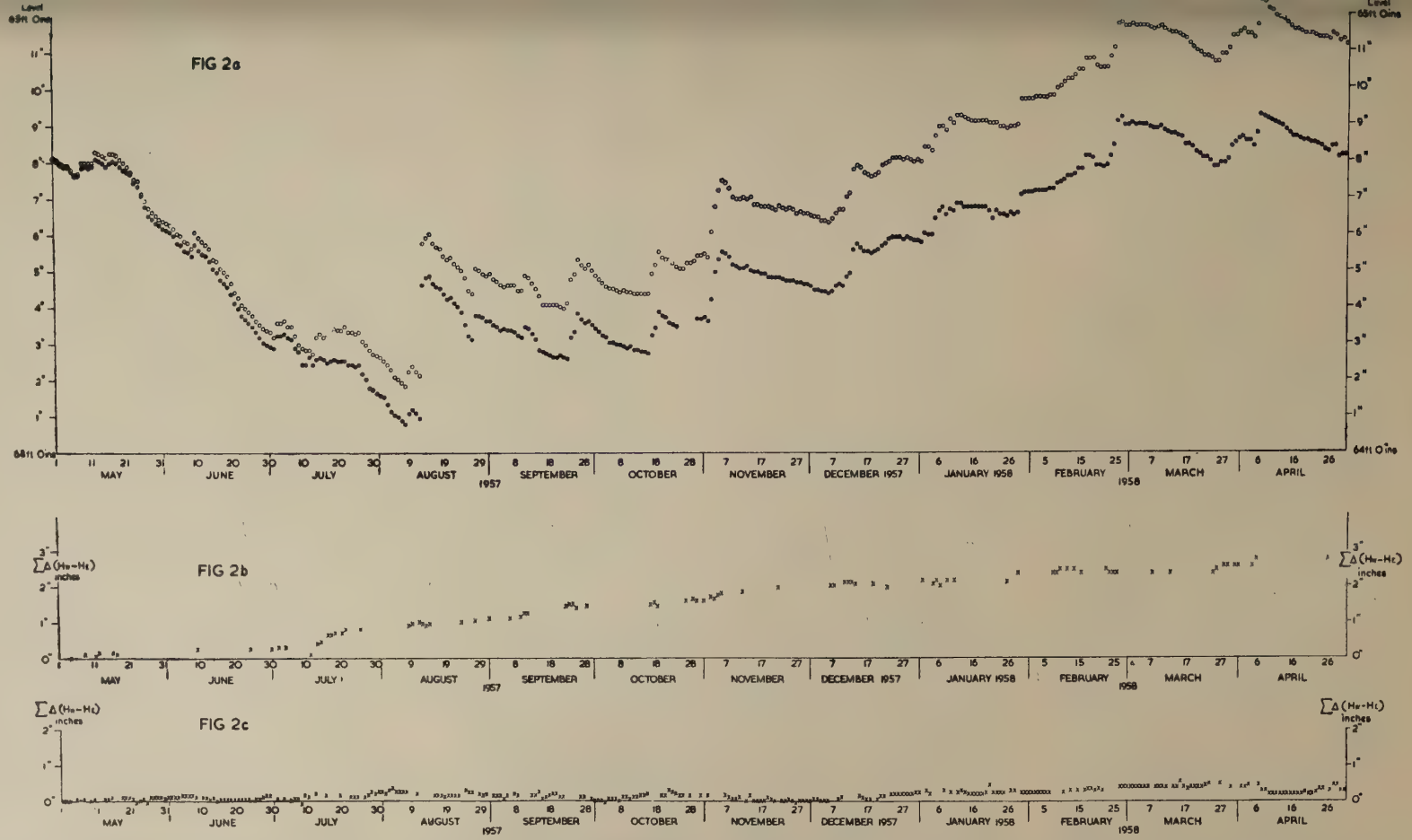

Figure 2. (a) Level of Kempton Park East Reservoir (●) and of Kempton Park West Reservoir (o), May 1957 to April 1958. The level gauges were set to the same reading on 1 May 1957, when the level of the East Reservoir was 64 ft. 8·1 in. above O.D. and that of the West Reservoir was about one foot higher. (b) and (c) Cumulative daily differences of the difference in level between Kempton Park East and West Reservoirs; (b) wet days, i.e., those having more than 0·05 in. rain; (c) dry days, i.e., those having 0·05 in. rain or less.

1. Observations

The two reservoirs concerned are the Kempton Park East and the Kempton Park West which were used by kind permission of the Chief Engineer of the Metropolitan Water Board. The general shape and situation of the reservoirs can be seen on Fig. 1. They are of similar construction and both are about 20 ft deep, but the East Reservoir is 40 acres in area and the West Reservoir 20 acres. Both were sealed as carefully as possible and no water was pumped in or out.

Water levels were measured daily to the nearest 0·05 in. by Metropolitan Water Board staff, the East Reservoir observations being made for a sub-committee of the Institution of Water Engineers and the West Reservoir observations being made at the request of the National Institute of Oceanography.

The observations from May 1957 to April 1958 are shown in Fig. 2 (a), from which it is clear that the level of the West Reservoir rose relative to that of the East Reservoir by three inches in the twelve months. This is about one-tenth of the annual evaporation. The annual evaporation and the annual rainfall are approximately equal.

2. Discussion

This is a surprising result for reservoirs of this area which are so close together in such flat country. No leaks or deficiencies in the measuring equipment were found.

The difference in level can be shown to be linked with the rainfall by using cumulative daily differences of the water-level difference on wet days and dry days separately. Formally, if H_E and H_W are the levels of the East and West Reservoirs respectively, then we take daily differences $\Delta (H_W - H_E)$ and form cumulative sums $\Sigma \Delta (H_W - H_E)$ for wet days and for dry days. For this purpose a wet day was taken as one on which more than 0·05 in. of rain was reported from Kempton Park ' A ' raingauge and a dry day as one on which the Kempton Park reading was 0·05 in. or less.

Figs. 2 (b) and 2 (c) show the results. It seems clear that changes in relative level are associated with wet days.

The effect can be partially explained by the geometry of the reservoirs, as the ratio of the collecting area to the free-surface area is greater in the West Reservoir; but this only accounts for a small fraction of the observed effect.

The simplest hypothesis is that the West Reservoir gets, on average, more rain than the East. This does not seem impossible but many meteorologists appear to consider it extremely unlikely. Certainly it would be a large undertaking to confirm it by using conventional raingauges. The two or three gauges in the neighbourhood are said to agree fairly well, but no detailed analysis has been reported.

On the other hand, the importance of effects such as this to water engineers is considerable; it is hoped that this note will stimulate further discussion of the problem.

Correspondence

551.521.11 (729.87) : 551.521.12 (729.87)

Solar radiation and the duration of sunshine in Trinidad, West Indies

By G. W. Smith

There is an error in my paper of this title which appeared in the October 1959 issue. The regression analysis which led to my Eq. (5)

$$R = R_A\,(0{\cdot}20 + 0{\cdot}59\,n/N)$$

was performed treating n/N as the independent variable. In other words, Eq. (5) considers that R/R_A is known, and that it is required to estimate the value of n/N. This is not usually the case. The whole point of the equation is to be able to estimate values of radiant energy when the number of hours of sunshine (as measured by a Campbell-Stokes recorder) is known.

Recalculating my data treating n/N as the dependent variable, gives the equation

$$R = R_A\,(0{\cdot}27 + 0{\cdot}49\,n/N) \quad . \quad . \quad . \quad . \quad (1)$$

This is in much better agreement with the equation given by Cowan for Jamaica

$$R = R_A\,(0{\cdot}31 + 0{\cdot}49\,n/N) \quad . \quad . \quad . \quad . \quad (2)$$

and also agrees better with the value calculated by Glover and McCulloch (1958) of

$$R = R_A\,(0{\cdot}29 + 0{\cdot}52\,n/N) \quad . \quad . \quad . \quad . \quad (3)$$

I am greatly indebted to these gentlemen for pointing out my error.

Reference

Glover, J. and McCulloch, J. S. G. 1958 *Quart. J. R. Met. Soc.*, **84**. p. 172.

Soils Section, R.R.C.,
Imperial College of Tropical Agriculture,
St. Augustine, Trinidad, W.I.

Report of the Council

for the year 1959

Submitted to the Annual General Meeting, 20 April 1960

General

The Council is pleased to report that the progress reported last year has been maintained during the year 1959 with further considerable success accompanied, in spite of a slight reduction in the Membership, by a satisfactory improvement in the financial position and in the launching of new activities.

In the autumn, Council introduced as an experimental measure, a new type of lecture primarily for the benefit of sixth-form physics pupils. Professor P. A. Sheppard gave the first of these lectures when, on three successive evenings, he addressed specially invited audiences from schools in the London and neighbouring areas on the subject 'Heat phenomena in the atmosphere : the atmosphere as a stage for the demonstration of the laws of heat.' The object of these lectures was to illustrate how basic physical principles are applied to the study of meteorological processes.

The series of Evening Discourses, first introduced in 1958, was continued and again met with encouraging support.

The sixth two-day Summer Meeting was held at Durham University. Further details are given later in the report.

During the year the Carnegie Trustees generously agreed to make a grant to the Society for the encouragement and promotion of field studies. The Society has thus been enabled to plan courses covering a far wider range of field study than has been possible in the past.

The Society is greatly indebted to the Nuffield Trustees for a loan, free of interest, to undertake the reprinting of back numbers of the *Quarterly Journal*.

In February a Special General Meeting approved amendments to the By-laws, as a result of which, firstly, the grade of Associate was abolished as from 1 January 1960, all existing Associates automatically becoming Fellows : secondly, a new grade, Corporate Membership, was established in order to provide a convenient means whereby business concerns and other bodies might be able to support the Society : and thirdly, provision was made for the payment of reduced annual subscription by Fellows of long standing who have retired from paid employment. Details of the changes appeared in the *Quarterly Journal*, **85**, No. 364.

Council

From 1 January to 30 September the constitution of the Council was as recorded in the 1958 Annual Report.

From 1 October 1959 the following Council held office :

President

James Martin Stagg, C.B., O.B.E., M.A., D.Sc.

Vice-Presidents

Richard Mead Goody, M.A., Ph.D.
Howard Latimer Penman, M.Sc., Ph.D., F.Inst.P.
George David Robinson, Ph.D., F.Inst.P.
Percival Albert Sheppard, Hon.A.R.C.S., B.Sc., F.Inst.P.

Vice-President for Canada

Warren Lehman Godson, Ph.D.

Vice-President for Scotland

Robert Cranna, M.A., B.Sc.

Treasurer

Brian Cecil Vernon Oddie, B.Sc.

Secretaries

Arthur George Forsdyke, Ph.D., A.R.C.S. (Librarian)
Patrick John Meade, O.B.E., B.Sc.
John Stanley Sawyer, M.A. (Editor)
Richard Segar Scorer, M.A., Ph.D.

Councillors

John Alan Chalmers, M.A., Ph.D., F.Inst.P.
George Arthur Corby, B.Sc.
John Robert Dark Francis, B.Sc. (Eng.), A.I.C.E.
Frank Henry Winn Green, M.Sc.
Arthur Leonard Maidens, B.Sc.
Robert James Murgatroyd, Ph.D.
Paul Robert Owen, B.Sc., A.F.R.Ae.S.
Reginald Charles Rainey, Ph.D.
John Brodrick Rigg, M.A.
John Russell Thorp, O.B.E., M.A.
Charles Desmond Walshaw, M.A., Ph.D.
Henry Lionel Wright, M.A.

The Council met eight times during the year and there were thirty-three Committee Meetings.

Meetings

Eight Ordinary Meetings, two Special Meetings and three Evening Discourses were held in the Society's rooms during the year.

The Symons Memorial Lecture was given on 1 April by Mr. J. A. Ratcliffe, F.R.S., Reader in Physics at Cambridge University and Fellow of Sidney Sussex College, who took as his subject ' The highest parts of the ionosphere.'

The Annual General Meeting was held in the Society's rooms on 22 April when the Report of Council and the Annual Accounts for 1958 were adopted. Professor P. A. Sheppard delivered the Presidential Address, his subject being ' Education and research in meteorology in the United Kingdom.'

On 13 May the Society, in conjunction with Messrs. Decca Radar Ltd., held a reception in the Society's rooms in honour of the Commonwealth delegates to the Meteorological Conference.

The Summer Visit on 8 July was to the Hydraulics Research Station of the Department of Scientific and Industrial Research at Howbery Park, Wallingford, Berkshire, by the kind permission of the Director, Mr. F. H. Allen. Some thirty Members attended and spent an interesting afternoon being shown round the Main Hall which houses the laboratories and workshops, and seeing model investigations which were being carried out on behalf of public departments. Council expresses its appreciation of a most enjoyable occasion.

The Popular Lecture, now in its tenth year, was given by Mr. J. MacDowall on 30 November and 7 December on ' Meteorology in Antarctica.'

During the year four successful vacation courses were held in conjunction with the Field Studies Council: two at Preston Montford, one at Malham Tarn and one, a new course on oceanography, at Dale, Pembrokeshire.

Council records its thanks for the generous grant from the Carnegie United Kingdom Trust which enabled the Society to give adequate financial support to the Malham Tarn and Dale Courses run in conjunction with the Field Studies Council, and to launch two new courses on meteorology at the Midland Gliding Club and the London Gliding Club.

Summer Meeting, Durham, 17-19 September 1959

The sixth two-day Summer Meeting of the Society was held at The Durham Colleges, University of Durham, by kind permission of the Warden, Sir James Duff, and the Head of the Department, Professor G. D. Rochester.

The Meeting was formally opened on the evening of 17 September at a reception at St Mary's College by Professor and Mrs. G. D. Rochester and Dr. E. M. Williamson, the Principal of the College.

Two Symposia were held in the Department of Physics. Professor G. D. Rochester took the Chair at the first on 'Atmospheric electricity.' Papers were read by Dr. J. E. Maund on 'Point discharge currents from natural and artificial points'; by Dr. J. Adamson on 'Compensation for potential gradient changes in measurements of air-earth current'; by Dr. T. W. Wormell who read Professor/Doctor H. Israël's paper 'Atmospheric electrical agitation'; by Dr. C. J. Adkins on 'The small ion concentration and space charge near the ground'; and by Dr. M. Smiddy on 'Measurements of space charge in the lower atmosphere using double field mills.'

Professor P. A. Sheppard, President of the Society, took the Chair at the second Symposium on 'Climatic change.' Contributions were made by Dr. A. E. M. Nairn, Professor Gordon Manley and Mr. H. H. Lamb.

A visit by coach was made to the Roman Wall at Housesteads when Mr. R. P. Wright acted as guide to the party. On the Saturday afternoon the party made a conducted tour of Durham Castle and the Cathedral.

The Council records its thanks to Dr. A. G. Forsdyke, who acted as co-ordinator, and to Dr. J. A. Chalmers, Dr. W. C. A. Hutchinson and their colleagues at Durham whose efforts contributed so much to the success of the meeting.

Publications

The number of papers submitted for publication in the *Quarterly Journal* showed a slight increase. Papers submitted for publication in the year totalled 67 as against 62 in 1958 and 64 in 1957. Papers published during the year numbered 43. The average time between receipt of approved copy and publication in the *Journal* was about $3\frac{1}{2}$ months. The four numbers of the *Journal* in 1959 contained 462 pages, which compares with 493 in 1958.

Council is greatly indebted to the Editor, Mr. J. S. Sawyer, the Editing Committee and the proof reader, Mr. E. L. Hawke, for their work in the production of the *Journal*.

Council has initiated a programme of reprinting back numbers of the *Quarterly Journal* where stocks are either completely or almost exhausted. Seven numbers have been reprinted; six more are in the hands of the printers, and when these reprinted Journals are received the Society will have at least fifteen copies of all volumes published since 1945.

Council records its thanks to Cmdr. J. R. Thorp and his Committee to whose hard work the continued success of the monthly magazine, *Weather*, is due. The Committee is to be specially congratulated on the steps taken to achieve publication during the period of the printing dispute at the end of the summer.

Membership

	Fellows and Foreign Members			Corporate Members	Associates	Students	Honorary Members	Total
	Annual	Life	Total					
31 Dec. 1958	1,407	173	1,580	—	31	101	16	1,728
Since elected	83		83	9		18		110
Re-admitted	6		6					6
Transferred	15		15		− 1	− 14		—
Deceased	− 8	− 6	− 14				− 1	− 15
Resigned	− 56		− 56		− 1	− 4		− 61
Struck off	− 54		− 54			− 8		− 62
31 Dec. 1959	1,393	167	1,560	9	29	93	15	1,706
Canadian Branch								
31 Dec. 1958	72	5	77		268			345
31 Dec. 1959	71	5	76		258			334
Grand Total								
31 Dec. 1958	1,479	178	1,657		299	101	16	2,073
31 Dec. 1959	1,464	172	1,636	9	287	93	15	2,040

Deaths

Council records with sorrow the deaths of Professor C. T. R. Wilson (1903), an Honorary Member and a Nobel Laureate, and of Mr. W. M. Witchell, Life Fellow (1924), who was for many years an Honorary Officer of the Society.

The Council also records with sorrow the deaths of W. R. G. Atkins, Life Fellow (1939); J. C. Beechey, Fellow (1927); H. H. Bentley, Life Fellow (1898); W. H. Elder, Fellow (1930); S. S. Elliott, Life Fellow (1911); W. T. Gillam, Life Fellow (1953); Mrs. A. Hawney, Fellow (1953); P. W. May, Life Fellow (1916); Guy Michell, Fellow (1930); E. Satterthwaite, Fellow (1951); J. N. Saunders, Fellow (1942); D. H. Thomson, Fellow (1933); R. F. Westaway, Fellow (1947).

Library

Loans of books and lantern slides have continued on much the same scale as in previous years but there has been an increased demand for films and filmstrips. Catalogues of the lantern slides and of the film and filmstrips were completed and have been in demand.

Ten additional series of journals and memoirs from different countries have been received. In two cases *Weather* is supplied in exchange and in once case the *Quarterly Journal*. Three lists of Accessions to the Library and one addendum to the catalogue of journals and memoirs were issued.

A further selection of the older journals and books has been rebound. To assist in the better preservation of the books an additional section of the open shelves has been fitted with glass panels.

' Master ' whole-plate prints of selected negatives in the Cave Collection of cloud photographs have been made. As a safeguard in the event of damage the negatives and prints are housed in different locations. Arrangements are in hand for the preparation of another set of cloud photographs in postcard form from the Clarke Collection because the demand for them continues.

The Society has received for retention in the Library a valuable folio of cloud sketches by Luke Howard, F.R.S. They were bequeathed to the Royal Meteorological Society by his granddaughter, Lady Fry.

Staff

During the year Mrs. A. Wallis, the Accounting Assistant, resigned her appointment after nearly 8 years' service. On the recommendation of the Officers, Council appointed Mr. J. H. Rogers, B.Comm. (London), to fill the vacancy on a part-time basis.

Other members of the staff consisted of Mr. R. H. Levinson, M.A., F.C.A., Assistant Secretary; Mr. R. S. Read, I.S.O., M.A., B.Sc., A.R.C.S., F.Inst.P., Assistant Librarian; Mrs. W. A. Steward, Editorial Assistant and Publications Clerk; Miss D. E. Hall, Secretarial Assistant; Miss E. Crafts, *Weather* Clerk; Miss E. Hooper, General Clerk.

The Officers and Council have pleasure in recording their high appreciation of the work of the staff during the past year.

Branch and Centres

Canadian Branch

Four meetings were held in Toronto during the year, in addition to the Annual Business Meeting in May. At the Annual Business Meeting the Executive of the Canadian Branch was transferred to Montreal. This Executive Committee consisted of President, Dr. J. S. Marshall; Vice-President, Mr. J. M. Leaver; Vice-President for Canada, Dr. W. L. Godson; Treasurer, Mr. H. M. Hutchon; Recording Secretary, Mr. B. W. Boville; Corresponding Secretary, Mr. R. Anderson; Councillors-at-Large, Chairman of Montreal Centre, Chairman of Winnipeg Centre, Chairman of Toronto Centre, Mr. P. Johns, Mr. C. H. Sutherland, Mr. C. E. Thompson, Mr. A. W. Jackson.

Montreal Centre

Seven meetings were held during the year on topics varying from 'Radioactive fallout' to 'Numerical weather prediction.' In addition to capable local speakers the Centre was fortunate to have such visiting speakers as Dr. M. Tepper, Chief, Severe Local Storms Research Unit, U.S. Weather Bureau, and Mr. R. List of Switzerland. The Annual General Meeting was held in March at which the following officers were elected: Chairman, Dr. S. Orvig; Secretary, Mr. R. Strachan and Treasurer, Mr. P. Summers.

Winnipeg Centre

Four meetings were held during the year. These meetings were well attended and there was a spirit of interest and enthusiasm evident at all of them. The programme of collecting and distributing meteorological periodicals and books to interested libraries in Greater Winnipeg was continued. The Officers for the year 1959 were: Honorary Chairman, Mr. A. B. Lowe; Chairman, Mr. A. H. Lamont; Vice-Chairman, Mr. P. Kowal; Secretary-Treasurer, Mr. H. M. Fraser.

Toronto Centre

Following the transfer of the Canadian Branch Executive to Montreal in May 1959 a Toronto Centre was established with the following officers: Chairman, Mr. M. K. Thomas; Secretary, Mr. A. G. MacVicar; Treasurer, Mr. H. E. Chadburn. Two meetings of the Toronto Centre were held during the autumn of 1959.

Scottish Centre

Four meetings were held during the year at Edinburgh University, and an excursion was made in May to the Building Research Station at Thorntonhall and the Civil Airport, Renfrew.

The Committee consisted of: Chairman and Vice-President for Scotland, Mr. J. R. Atkinson, succeeded in October by Mr. R. Cranna; Secretary and Treasurer, Mr. D. H. McIntosh; and eleven other Fellows.

Manchester Centre

The membership of the Centre remains as previously at twenty-four. Attendances at the five meetings held during the year were slightly less than in 1958. There was an audience of about fifteen at most meetings of whom some were guests of Centre Members. The best attended meeting was in May when Professor R. Fleagle from Washington gave an address on 'The general circulation.'

A very successful Popular Lecture to some 120 sixth-form boys and girls was given on 10 December by Mr. J. MacDowall on 'Meteorology in Antarctica.' This meeting was held in the Reynolds Hall of the College of Science and Technology in Manchester with the co-operation of the Manchester Education Committee.

The Centre has been as in former years a constituent body of the Manchester Federation of Scientific Societies.

During the year Professor M. J. Lighthill ceased to be a Member of the Committee on his departure from Manchester to take up the post of Director, The Royal Aircraft Establishment, Farnborough.

Representatives

The Society's representatives on other scientific bodies were :

Royal Society Committees
Gassiot Committee, President; *British National Committee for Geodesy and Geophysics*, Dr. R. C. Sutcliffe, Dr. T. W. Wormell; *British National Committee for Physics*, Dr. F. Pasquill; *British National Committee for Oceanic Research*, Dr. T. H. Ellison; *National Committee for Antarctic Research*, Mr. H. H. Lamb; *National Committee for Space Research*, Professor P. A. Sheppard.

Institute of Physics Advisory Committees
Journal of Scientific Instruments, Dr. H. L. Penman; *British Journal of Applied Physics*, Dr. G. D. Robinson.

Cambridge University Meteorological Committee
Professor P. A. Sheppard.

Royal Astronomical Society
Geophysical Committee, Mr. J. S. Sawyer.

Advisory Committee on Meteorology for Scotland
Sir Charles Normand (up to December 1959); Professor P. A. Sheppard (from December 1959).

British Glaciological Society
Professor Gordon Manley.

Awards

The following awards were made at the Annual General Meeting on 22 April :

Symons Memorial Gold Medal for 1959

Sir Graham Sutton, C.B.E., F.R.S., Director-General of the Meteorological Office, London, for his outstanding work in the field of micrometeorology.

Napier Shaw Memorial Prize

The second award of this prize, instituted in 1954 to mark the centenary of the birth of Sir Napier Shaw, was awarded to Professor S. K. Runcorn of the Department of Physics, Newcastle (University of Durham), for his essay, ' Climatic change through geological time in the light of the palaeomagnetic evidence for polar wandering and continental drift.' The award consisted of a prize of £100 and was announced by the President at the Ordinary Meeting of the Society of 13 May 1959.

Darton Prizes for 1958

Home Awards :
1st Prize : Mr. R. J. Taylor.
2nd Prize : Mr. D. D. Clark.

Canadian Awards :
1st Prize : Mr. J. L. Galloway.
2nd Prize : Mr. R. W. Longley.

Scientific Activities Fund

During the year the following grants were made :

Mr. T. L. Hirst towards the cost of attending the Meteorological Course at Malham Tarn Field Centre.

Mr. W. S. Hall towards to the cost of attending the Meteorological Course at Malham Tarn Field Centre.

Mr. J. A. Taylor to assist research work and meteorological experiments in South Lancashire.

Finance

There was a surplus for the year of £1,755, compared with £919 in 1958. This increase is, however, more than accounted for by a refund of income tax on covenanted subscriptions, which we received in 1959 but which was due in respect of subscriptions for 1956 and 1957 : payment has been delayed by certain legal proceedings.

Few items in the Income and Expenditure Account differ greatly from the previous year's, or call for special comment. Total expenditure was lower than in 1958 by about £250 but income (apart from the tax refund already mentioned) also decreased. The decrease in subscriptions of £156 is perhaps the most disquieting feature, since it reflects a small decline in membership. On the other hand, it is gratifying to see increases in the sales of both of the Society's publications : *Weather* has had a particularly successful year, increasing both its sales and its revenue from advertising by over 14 per cent; great credit is due both to the Editors and to the *Weather* Clerk.

In the Balance Sheet, the only item which calls for comment is the loan free of interest, which has been made to the Society by the Nuffield Foundation to enable it to reprint back numbers of the *Quarterly Journal*. Council acknowledges with gratitude this generous assistance. As noted elsewhere, the reprinting is already in hand.

The Society's investments did well during the year, their total value increasing by £5,142. About £605 of this represents new investment, the remainder being due to rises in value of the stocks.

It must be expected that the Society's expenses over the next few years will be greater than in 1959. The Society will have to pay for the reprinting of back numbers of the *Quarterly Journal* and for the translation of the Russian periodical, *Works of the Central Aerological Observatory, Moscow*, – two enterprises of which the costs have not yet been fully felt. Moreover, the costs of materials and services of all kinds, which have been fairly steady of late may well begin to rise once more. Nevertheless, the present financial position of the Society is satisfactory and we have no reason to fear any serious financial difficulties in the foreseeable future.

RECEIPTS AND PAYMENTS OF SPECIAL FUNDS FOR 1959

	Balance at 1.1.59	Income for 1959	Expenditure for 1959	Balance at 31.12.59
	£	£	£	£
Awards Fund	6	—	—	6
British Rainfall Organization Fund ..	1 (Dr.)	200	113	86
Life Composition Fund	91	114	155 (a)	50
Napier Shaw Memorial Fund	259	48	100 (b)	207
Royal Society Reprints Scheme	14	—	12	2
Scientific Activities Fund	67	174 (c)	85	156
Staff Pension Fund	536	595 (d)	1,236 (e)	105 (Dr.)
Symons Memorial Fund	44	36	46	34
Hampton Brown Fund	30	60	—	90
	£1,046	£1,227	£1,747	£526

(a) Made up of : £100 transferred to " A " Account. £55 Purchase of 3½ Royal Dutch Petroleum Shares. (b) 1959 Prize. (c) Includes £100 transferred from " A " Account. (d) Includes transfer of 15% of pensionable Salaries £390. (e) Made up of : £50 Purchase of £20 English Electric Co. Ltd. Ordinary Stock. £1,186 Pension set aside for former Assistant Secretary.

Examined and found correct.
3 March 1960

BALL, BAKER, DEED & CO.,
Chartered Accountants.

VALUATION OF INVESTMENTS AT 31 DECEMBER 1959

Market Value 31.12.58			*Market Value 31.12.59*	
£	£		£	£
		*General Fund**		
	912	£1,250 4% Consolidated Stock 1957 or after x.d.	912	
	181	£200 4% Funding Stock 1960/90	187	
	555	£649.1.7 Government of Commonwealth of Australia 4% Registered Stock 1955/70	584	
	—	£500 5¾% Hawker Siddeley Conv. Deb. Stock 1979/84	547	
	400	£200 Burmah Oil Co. Ltd. Ordinary Stock	550	
	832	£287 Guest Keen & Nettlefold Ltd. Ordinary Stock	1,309	
	345	£200 Courtaulds Ltd. Ordinary Stock	667	
	555	240 Marks & Spencer Ltd. "A" Ordinary 5/– Shares	912	
	410	£160 Babcock & Wilcox Ltd. Ordinary Stock	390	
	450	£250 Vickers Ltd. Ordinary Stock	494	
	576	£131 Unilever Ltd. Ordinary Stock	925	
	512	£130 John Dickinson & Company Ltd. Ordinary Stock	699	
	628	£150 Tube Investments Ltd. Ordinary Stock	1,022	
6,356				9,198
		Scientific Activities Fund		
	166	£250 3½% War Stock 1952 or after	163	
	66	£100 3½% Conversion Loan 1961 or after	65	
	800	£1,000 4% Trinidad Government Stock 1973/76	800	
1,032				1,028
		Staff Pensions Fund		
	657	£900 4% Consolidated Stock 1957 or after x.d.	657	
	400	£500 4% Trinidad Government Stock 1973/76	400	
	465	£600 3½% Treasury Stock 1979/81	474	
	522	£500 Bowater Paper Corporation Ltd. 5¾% Convertible Unsecured Loan Stock 1978/82	715	
	612	£310 English Electric Co. Ltd. Ordinary Stock	866	
	574	£150 Turner & Newall Ltd. Ordinary Stock	829	
	162	£150 British Petroleum Co. Ltd. 6% Convertible Debenture Stock 1976/80	158	
	503	£375 Distillers Company Ltd. Ordinary 10/– Shares x.d.	731	
	386	£300 Beecham Group Ltd. Ordinary 5/– Shares	723	
4,281				5,553
		Life Composition Fund		
	255	£350 4% Consolidated Stock 1957 or after x.d.	255	
	775	£1,000 3½% Treasury Stock 1979/81	790	
	522	£500 Bowater Paper Corporation Ltd. 5¾% Convertible Unsecured Loan Stock 1978/82	715	
	570	£300 Imperial Chemical Industries Ltd. Ordinary Stock	915	
	279	£100 Canadian Eagle Oil Company Ltd. Ordinary Shares n.p.v.†	—	
	—	£25 "Shell" Transport & Trading Co. Ltd. Ord. Stock (Reg.)†	200	
	—	20 Royal Dutch Petroleum Co. fl.20 Shares (Bearer)†	333	
2,401				3,208
		British Rainfall Organization Fund		
	499	£750 3½% War Stock 1952 or after	491	
	2,060	£3,097.10.0 British Transport 3% Guaranteed Stock 1978/88	2,091	
	73	£100 4% Consolidated Stock 1957 or after x.d.	73	
	222	£300 3½% Treasury Stock 1979/81	237	
	1,790	£2,000 4% Government of Commonwealth of Australia Registered Stock 1966/68 x.d.	1,850	
4,644				4,742
		Napier Shaw Memorial Fund		
	233	£350 3½% Conversion Loan 1961 or after	229	
	292	£400 4% Consolidated Stock 1957 or after x.d.	292	
525				521
		Symons Memorial Fund		
	432	£650 3½% Conversion Loan 1961 or after	425	
	146	£200 4% Consolidated Stock 1957 or after x.d.	146	
578				571
		Hampton Brown Fund		
	800	£500 Hawker Siddeley Group Ltd. Ordinary £1 Shares	734	
	147	375 Marley Tile (Holding) Co. Ltd. 'A' Ordinary non-voting 5/– Shares	351	
947				1,085
£20,764				£25,906

* General Fund also includes the Freehold of 49 Cromwell Road, S.W.7.

† In exchange for its holding of Canadian Eagle Oil Company Ltd. shares the Fund received £25 "Shell" Transport and Trading Company Ltd. Ordinary Stock (Reg.) and 16⅔ Royal Dutch Petroleum Company. Fl.20 Shares (Bearer). An option to purchase an additional 3⅓ of the latter was exercised.

The Society also holds £750 National Savings Certificates and £400 Building Society Shares which have been set aside for a former Assistant Secretary's Pension.

Examined and securities seen.
3 March 1960

BALL, BAKER, DEED & CO.,
Chartered Accountants

GENERAL

BALANCE SHEET AT

31.12.58 £	£	£		£	£	£
			ACCUMULATED FUND—			
	12,111		Excess of Assets over Liabilities at 1 January 1959		12,691	
	619		*Deduct* Library Equipment Fund		683	
	11,492				12,008	
	340		*Deduct* Loss on Sale of Investments		—	
	11,152				12,008	
	919		*Add* Excess of Income over Expenditure		1,755	
12,071						13,763
619			LIBRARY EQUIPMENT FUND			683
350			REPAIRS RESERVE FUND			556
—			NUFFIELD FOUNDATION – LOAN UNSECURED			1,966
			CURRENT LIABILITIES—			
	1,238		Creditors and Accrued Charges		960	
			Amounts Received in Advance :			
		1,289	Publications	1,767		
		329	Subscriptions	469		
	1,618				2,236	
2,856						3,196

J. M. STAGG—*President*

B. C. V. ODDIE—*Treasurer*

We have examined the above Balance Sheet and annexed Income and Expenditure Account which in our opinion respectively give a true and fair view of the state of the Society's affairs as at 31 December 1959, and of its income and expenditure for the year ended on that date.

Lloyds Bank Buildings,
55/61, Moorgate,
London, E.C.2.
26 April 1960

BALL, BAKER, DEED & Co.,
Chartered Accountants

£15,896

£20,152

FUND

31 DECEMBER 1959

31.12.58						
£	*£*				*£*	*£*
		FIXED ASSETS—				
5,375		Freehold Property – Estimated value (in 1925) of the Society's House, 49 Cromwell Road				5,375
		Furniture and Equipment—				
			At 1.1.59	Additions during year	Accumulated Depreciation	Net
1,143		General	1,699	127	692	1,134
619		Library	824	126	267	683
1,762			£2,523	£253	£959	£1,817	1,817
		(No amount has been included for the value of the Library and the stock of publications)				
		INVESTMENTS—				
		Valued at Market Price 31.12.54 with additions since at cost—				
1,206		£1,250 4% Consolidated Stock 1957 or after ..			1,206	
206		£200 4% Funding Stock 1960/90			206	
645		£649.1.7 Government of Australia 4% Registered Stock 1955/70			645	
—		£500 5¾% Hawker Siddeley Conv. Deb. Stock 1979/84			500	
562		£200 Burmah Oil Co. Ltd. Ordinary Stock ..			562	
612		£287 Guest, Keen & Nettlefolds Ltd. Ordinary Stock			612	
368		£200 Courtaulds Ltd. Ordinary Stock			368	
506		240 Marks & Spencer Ltd. "A" Ordinary 5/– Shares			506	
396		£160 Babcock & Wilcox Ltd. Ordinary Stock ..			396	
392		£250 Vickers Ltd. Ordinary Stock			392	
392		£131 Unilever Ltd. Ordinary Stock			391	
414		£130 John Dickinson & Co. Ltd. Ordinary Stock			414	
425		£150 Tube Investments Ordinary Stock ..			425	
6,124						6,623
		(Market Value 31.12.59 £9,198)				
		(Market Value 31.12.58 £6,356)				
		CURRENT ASSETS—				
495		*Weather* Paper Stock				295
		Debtors—				
	8	*Proceedings Toronto Meteorological Conference 1953*			—	
	527	Subscriptions			1,415	
	815	Publications			1,202	
	1,350				2,617	
	250	*Less* Provision for Bad and Doubtful Debts ..			425	
	1,100				2,192	
	—	Dividend Tax Reclaim 1958/59			242	
	25	Payments in Advance			52	
	50	Money on Deposit			2,823	
	965	Balance at Bankers and Cash in Hand			733	
2,140						6,042
£15,896						£20,152

GENERAL

INCOME AND EXPENDITURE ACCOUNT

EXPENDITURE

1958 £	£		£	£
		Establishment—		
26		Water Rates	28	
275		Cleaning Wages and Materials	308	
24		Insurance	30	
248		Lighting and Heating	214	
271		Repairs and Maintenance	262	
350		Transfer to Repairs Reserve Fund	200	
127		Depreciation of Furniture and Equipment	136	
1,321				1,178
		Administration—		
3,460		Salaries	3,420	
450		Contribution to Staff Pension Fund	390	
122		Printing	146	
374		Stationery	369	
1,003		Postage, Carriage and Packing	977	
87		Telephone	93	
		Audit and Accountancy Fees—		
136		Royal Meteorological Society and *Weather* Magazine	136	
14		Legal Expenses	—	
77		Sundries	74	
5,723				5,605
		Publications—		
3,948		*Quarterly Journal*	3,637	
2,394		*Weather*	2,526	
284		Illustrations	381	
132		*Cloud Study*	26	
103		Reprints of *Weather*	126	
214		Reprints of Papers	256	
34		*Monthly Weather Report*	35	
—		Translation Russian Journal	51	
24		Secondhand Journals	30	
7,133				7,068
		Library—		
491		Salary	491	
210		Books, Bindings, etc.	226	
62		Depreciation of Equipment	62	
763				779
		Meetings' Expenses—		
67		Travelling Expenses	82	
105		London	92	
40		Scotland	40	
30		Manchester	30	
20		*Weather* Meetings	32	
262				276
128		**Provision for Bad and Doubtful Debts**		175
		Grants and Prizes—		
100		Scientific Activities Fund	100	
40		Grant to Society for Promotion of Field Studies	40	
140				140
919		**Balance, being Excess of Income Over Expenditure**		1,755
£16,389				£16,976

FUND

FOR THE YEAR ENDING 31 DECEMBER 1959

INCOME

1958 £	£	£		£	£	£
			SUBSCRIPTIONS, ETC.—			
	7,160		Subscriptions		7,004	
	96		Entrance Fees		81	
	18		Donations		6	
	7,274				7,091	
	145		Less Adjustments for Lapsed Membership (By-Law 40)		236	
7,129						6,855
—			Tax recovered under Deed of Covenant 1955/56 and 1956/57			898
			PUBLICATIONS—			
	3,856		Sale of Publications – Current *Q.J.*		4,005	
	750		Sale of Publications – back numbers *Q.J.*		347	
	1,476		Sale of *Weather* – excluding members' copies		1,691	
	294		Sale of *Cloud Study*		79	
	267		Reprints, *Q.J.*		336	
	113		Reprints, *Weather*		139	
	525		Advertisements, *Q.J.*		555	
	915		Advertisements, *Weather*		1,051	
	8		*Proceedings Toronto Meteorological Conference 1953*– Sales etc.		—	
	24		Sundry Sales		6	
8,228						8,209
			INVESTMENT INCOME—			
	191		Interest on Government Stocks (Gross)		124	
	100		Interest on Industrial Securities (Gross)		191	
	210		Rent of Maisonettes		495	
501						810
			PHOTOGRAPHS—			
	8		Sale of Cloud Postcards		5	
		142	Clarke and Cave Copyright Fees	84		
		44	*Less* Reproduction Costs	—		
	98				84	
106						89
			LIBRARY—			
200			Grant from the Royal Society		—	
			AMOUNTS RECEIVED FROM OTHER FUNDS—			
	200		Life Composition Fund		100	
	15		British Rainfall Organization Fund (for Administration)		15	
215						115
10			Profit on Sale of Typewriter		—	
£16,389						£16,976

Proceedings at meetings of the Society

Ordinary Meeting, 16 March 1960

The following candidates were declared to have been elected that day by the Council :

Fellows

Cook, John Kyle, c/o Nigerian Ports Authority, Private Post Bag 2588, Lagos, Nigeria, W. Africa.
Dearing, Ernest, B.Sc., 37 Ormerod Road, Burnley, Lancs (*reinstatement*; *elected 1946, resigned 1952*).
Hofmeyr, Wynand Louw, M.Sc., D.Sc., Chief Meteorologist, Weather Bureau, Private Bag 97, Pretoria, S. Africa.
Piéjus, Sqdn. Ldr. Philip C., 14 Witfontein Road, Heatherlands, George, Cape Province, S. Africa (*reinstatement*; *elected 1942, resigned 1950*).
Shepstone, Basil John, B.Sc., M.Sc., D.Sc., A.Inst.P., M.Inst.P. (S.A.), F.R.A.S., F.C.S., Exeter College, Oxford.
Stafford, Margaret Molly, B.A., 100 Chatsworth Avenue, Cosham, Portsmouth, Hants.

Foreign Members

Baker, Donald Gardner, B.S., M.S., Ph.D., Department of Soils, University of Minnesota, St. Paul 1, Minnesota, U.S.A.
Dessens, Henri J. J., Directeur de l'Observatoire du Puy de Dôme, 12 Avenue des Landais, Clermont Ferrand (Puy de Dôme), France.
Epstein, Edward S., A.B., M.B.A., M.S., Ph.D. (Pennsylvania State University), 1738 Barrington Place, Ann Arbor, Michigan, U.S.A.

Student Members

Griggs, Michael, The Clarendon Laboratory, Parks Road, Oxford.
Hanson, William, 93 Falconwood Avenue, Welling, Kent.

The following papers were then read and discussed :

1. ' Synoptic and dynamic aspects of climatic change ' by E. B. Kraus (*Q.J.*, **86**, p. 1), presented by Dr. E. T. Eady.

2. ' A synoptic study of day-to-day changes of ozone over the British Isles ' by D. W. Martin and A. W. Brewer (*Q.J.*, **85**, p. 393), presented by Dr. D. W. Martin.

Annual General Meeting, 20 April 1960

Dr. J. M. Stagg, C.B., O.B.E., M.A., D.Sc., President, in the Chair. The minutes of the Annual General Meeting held on 22 April 1959 were confirmed and signed.

The President in presenting the Report of Council for 1959 referred to the main items of interest.

A new series of sixth-form lectures had been instituted during the year. Professor Sheppard had given the first of this series to three separate audiences, taking as his subject, ' Heat phenomena in the atmosphere : the atmosphere as a stage for the demonstration of the laws of heat.'

Three Evening Discourses had been given during the year and all were well attended.

The President drew special attention to the large number of meetings, apart from Ordinary monthly meetings, sponsored by the Society.

Special reference was made to the success of the Vacation Courses in field study, sponsored by the Society in collaboration with the Field Studies Council, the success of which was now assured by a generous grant from the Carnegie United Kingdom Trust and by the enthusiasm of Dr. Scorer and the panel of instructors and lecturers.

The sixth two-day Summer Meeting was held at Durham in the Department of Physics, The Durham Colleges, University of Durham.

The Summer Visit was to the Hydraulics Research Station of the Department of Scientific and Industrial Research, Wallingford, Berkshire.

The *Quarterly Journal* had maintained its very high scientific standard and international reputation.

Weather had maintained a lively variety of articles and a high general standard throughout the year. For the first time, sales in 1959, excluding members' copies, exceeded 18,000 copies.

In reviewing the Membership of the Society, the President drew attention to the new grade of Corporate Membership and also to the abolition of the Associate Grade. Both changes took effect as from 1 January 1960.

The President referred to the increased demands being made for loans of books, lantern slides, etc., from the Library, but considered that still more use could be made of the facilities available.

Before calling on the Treasurer to present his report, the President expressed thanks to all those who sacrificed so much of their private time in the interests of the Society and also referred with appreciation to the services of the Staff during the year.

The Treasurer, Mr. B. C. V. Oddie, reported that the Society had had a satisfactory year financially, and £898 had been recovered during the year by way of rebate of income tax on covenanted subscriptions in respect of the years 1955/56 and 1956/57. An appeal had recently been made to Members to enter into deeds of covenant for payment of their subscriptions with a view to enhancing the real value of the amount actually paid by the Members.

During the year the Society had received from the Nuffield Foundation a loan of £963 for reprinting back numbers of the *Quarterly Journal*.

The Society's investments had done well during the year and the value of its holding of industrial shares had risen by 50 per cent.

Expenses must be expected to increase during the next two years and arrears of decorating had accumulated, but in spite of these additional liabilities the outlook was satisfactory.

The Treasurer then referred to By-law 86, which restricted the investment during any year in non-Trustee stocks to an amount approved by Council and authorised by a resolution passed at the preceding Annual General Meeting. During the past year investments had been made chiefly in taking up bonus issues to the extent of £717, and he asked for retrospective approval for this action. The proposal was duly seconded by Mr. J. M. Craddock, and unanimously approved.

The Treasurer also proposed that a sum of not more than £750 should be approved for investment during the coming twelve months. This motion was also seconded by Mr. J. M. Craddock and unanimously approved.

The President moved that the Report be received and adopted and that it be printed in the *Quarterly Journal*. The motion was carried unanimously.

Dr. F. Pasquill proposed a vote of thanks to the President and Council for their work during 1959 and this was seconded by Mr. M. E. Shoenberg.

The President then announced the following awards :

(a) *Buchan Prize* in respect of the period 1955–1959 :

To Robert James Murgatroyd, Ph.D., for his outstanding contributions to the study of the physical processes of the atmosphere.

(b) *Hugh Robert Mill Medal and Prize for 1959* :

To Basil John Mason, M.Sc., D.Sc., by whose acute insight into the physics of nucleation of cloud particles and exceptional experimental skill contributions have been made to the knowledge of processes that lead to cloud formation and to the development of precipitation.

(c) *Darton Prizes for 1959* :

Home Awards :

1st Prize : Dr. C. J. Adkins for his papers ' The small ion concentration and space charge near the ground ' (*Q.J.*, **85**, p. 237, 1959) and ' A rate of rainfall recorder ' (*Q.J.*, **85**, p. 419, 1959).

2nd Prize : Dr. J. D. Funk for his paper ' An improved polythene-shield net radiometer ' (*J. Sci. Inst.*, June 1959, p. 267).

Canadian Awards :

1st Prize : Dr. R. H. Douglas and Dr. W. Hitschfeld for their paper ' Patterns of hail-storms in Alberta ' (*Q.J.*, **85**, p. 105, 1959).

2nd Prize : Mr. D. K. A. Gillies for his paper ' Winds and water levels on Lake Erie ' (*Canad. Branch R. Met. Soc. Pap.*, **9**, No. 1, p. 12, 1959).

Dr. J. M. Stagg, C.B., O.B.E., M.A., D.Sc., then delivered the Presidential Address.

Dr. H. L. Penman proposed a vote of thanks to the President for his Address and asked that he would allow it to be printed in the *Quarterly Journal*. Mr. J. R. D. Francis seconded the vote of thanks.

The meeting was declared closed at 7 p.m.

Symons Memorial Lecture, 4 May 1960

Dr. Ragnar Fjørtoft, Director of the Norwegian Meteorological Office, Oslo, delivered the Symons Memorial Lecture, his subject being ' The general circulation of the atmosphere.'

Ordinary Meeting, 18 May 1960

The following candidates were declared to have been elected that day by the Council :

Fellows

Bailey, Capt. Trevor William, ' Westwood Marldell,' West Meon, Hants.
Bennion, Brian, 23 Wilton Extent, Heywood, Lancs.
Burton, James Michael Crowther, 5 Compayne Gardens, London, N.W.6.
Gill, Joginder Singh, c/o Nautical and Engineering College, Azad-Maidan, Bombay, India.
Laker, David Dalziel, c/o Barclays Bank Ltd., Chichester, Sussex.
Loewe, Fritz, Ph.D., Department of Meteorology, University, Parkville, N.2, Victoria, Australia (*transfer from Foreign Membership*).
Sen-Gupta, Bijoy Kumar, B.Sc., (Eng.), A.C.G.I., 5 Lyndhurst Gardens, London, N.W.3.
Singleton, Frank, B.Sc., D.I.C., Meteorological Research Flight, Royal Aircraft Establishment, Farnborough, Hants. (*transfer from Student Membership*).
Symonds, Kenneth Alan, F.R.G.S., 58 Nevin Drive, Chingford, London, E.4.

Foreign Member

Gilead, Mordekhai, P.O. Box 7036, Tel Aviv, Israel.

Student Member

Turner, Andrew Metters Russ, ' Jacota,' The Ridgeway, Fetcham, Leatherhead, Surrey.

Canadian Branch

Foreign Member

Schwarz, Uri, c/o Meteorological Section, International Civil Aviation Organization, 1080 University Street, Montreal, Quebec.

Fellows

Thacker, James Johnston, B.A.Sc., M.A., c/o Department of Transport, Goose Bay, Labrador
Verge, Robert William B.Sc., M.A., Department of Transport, Goose Airport, Labrador.

The following papers were then read and discussed :

1. ' The summer circulation of the arctic stratosphere below 30 km ' by F. K. Hare (*Q.J.* **86**, p. 127).
2. ' An experimental system for directly recording statistics of the intensity of atmospheric turbulence ' by J. I. P. Jones and F. Pasquill (*Q.J.*, **85**, p. 225), presented by Dr. F. Pasquill.
3. ' The reflection of short-wave radiation by vegetation ' by J. L. Monteith (*Q.J.*, **85**, p. 386).

Reviews

Glossary of meteorology. Ed. by Ralph E. Huschke. Boston (American Meteorological Society), 1959. Pp. viii, 638. $12.00.

There were three principal contributors and 38 ' subject area ' contributors to this Glossary, one of the principals being the late Dr. C. E. P. Brooks. Each of the 7,247 definitions represents the efforts of at least three individuals, at least two being specialists in the subject area involved.

The collecting of terms began in 1952 with an exhaustive survey of the literature of the previous five years. This list, plus previous collections (including M.O.225, *The Meteorological Glossary*), provided the platform for the final product. Some terms from sister disciplines such as hydrology, oceanography, etc., are included; some from basic sciences of physics, chemistry and from applicable portions of mathematics, statistics and electronics.

To quote from the Preface : the *Glossary of meteorology* purports to define every important meteorological term likely to be found in the literature today; it attempts to present definitions that are understandable to the generalist and yet palatable to the specialist; and it intends to be a reference book that satisfies its user in a minimum of time. It is the reviewer's opinion that these aims have broadly been met. The cross references in the text are useful, as are the references to the literature. What criticism the reviewer has must be read within this general appreciation of this quite massive work.

The World Meteorological Organization compiled in 1959 a provisional edition of the *International Meteorological Vocabulary*. No exhaustive comparison has been possible but there were to me significant differences in the definitions of Applied meteorology, Aerosol, Tropopause, to quote only three. I regard such differences as unfortunate because reader and writer should be dealing in the same terms. Again it is unhappy to find terms, e.g., centre jump, dewpoint formula, which are of the ' synoptic jargon '-type, included. This could be interpreted as endowing such terms with a wholly unwarranted status. Furthermore, there are defined such terms as beltane-ree, blad, piner, teuchit, peasweep – all of which come from these islands – but are they important beyond local boundaries ? One would hope that writers would not use such terms without defining them.

To sum up I would say that this work is a significant contribution in the field and undoubtedly will be much used. It is certainly a milestone in the progress towards a wholly acceptable document within which writer and reader can operate unambiguously.

R.H.C.

Annual Report on the Meteorological Office for the year ending 31 March 1959 (M.O.659). London (H.M. Stationery Office). Pp. 52; Tables; 5 Plates. 3s. 6d.

The Meteorological Office, as the latest Annual Report shows, is a national institution in which staff and tax-payer alike can continue to take legitimate pride. Taking advantage of the privilege of a review, however, three points, one particular and two general, may be selected for comment.

Five years ago a weather radar set was first installed on the Air Ministry roof; and, two years before that, the reviewer had happened to be watching a similar set in operation at the Entebbe Meteorological Office when a B.O.A.C. captain called in to express his appreciation of the guidance which it had provided among the cumulonimbus over Lake Victoria. However, it has subsequently not been very easy for one of the 8,000 daily diallers of WEA to detect analogous guidance on the distribution and movement of showers over Greater London. Misgivings on this point are not entirely dispelled by a reference in this year's Report to ' a first step ' in the regular operational use of the set on Victory House, in the form of experimental forecasts to London Transport. This cautious approach – and fading memories of AIRMET – tempt one to wonder whether the Meteorological Office and the B.B.C. may not contrive, by joint default, to leave to sponsored television the opportunity of introducing the British viewer to radar pictures of his own weather on his own screen.

While, with changes in the international scene, Meteorological Office personnel have been withdrawn from six overseas stations, staff have at the same time been seconded to the World Meteorological Organization and to the territorial meteorological services of ten countries, five of them in the tropics; and, at the recent W.M.O. Symposium on Tropical Meteorology, for example, one of these young meteorologists was joint author of an outstanding contribution on the longstanding problem of the relationship between pressure and wind near the Equator. New opportunities thus appear, to offset those lost by political changes; and one hopes that the scientific value and the professional stimulation of work in the tropics will continue to be appreciated in the Office.

The overall recruitment position continues to give cause for concern; and a temporary improvement reported during the previous year has not been maintained. Thus over the past eight years an 11 per cent net loss has been recorded in the scientific officer grades, despite the stimulus of new facilities such as those of the Meteorological Research Flight and, now, the Office's own high-speed digital computer. The depressing picture so suggested, with progressively fewer scientists exploiting an inevitably diminishing proportion of the potentialities of progressively improving facilities, may, one hopes, be at least partially untrue; but it would be consistent with the general failure of meteorology to attract its share of new minds, to which Professor Sheppard directed the attention of the Society in his 1959 Presidential Address. One ventures to suggest that this present inadequately competitive status of meteorology may to some extent reflect a lack of recognition,

even among some of its practitioners, that meteorology is characteristically concerned with the study of systems of a complexity approaching those of biology, for which the techniques of conventional mathematics and physics are tools which are necessary but not sufficient; demanding, as J. Bjerknes has emphasised, detailed studies of actual individual cases; and perhaps offering particular scope to the operational research approach. Might one in fact suggest that the stigma of ' inexact ' is perhaps less appropriate to meteorology than the challenge of ' complex '?

R.C.R.

Climatic Atlas of the Czechoslovak Republic. Prague (Hydro-Meteorological Institute), 1958. Pp. 107, 89 Maps; 11 Diagrams.

This very bulky volume contains 89 maps, 11 diagrams and a short explanatory text which together give what is described in the Preface as ' the first systematic and complete picture of the climate of the Czechoslovak Republic.' A supplement contains translations of the text into Russian, English and French. Although there is a good selection of the charts one would find in any climatic atlas, the contents suggest that in its compilation special attention was paid to the interests of agriculturalists.

Preliminary charts show the administrative departments of Czechoslovakia, then the topography, geology, forest regions and climatic regions of the country, these latter being based on criteria explained in the Introduction. There follows a long sequence of charts showing temperature, rainfall and sunshine data and various other climatic characteristics. Finally come twelve maps of phenological characteristics of certain crops and some diagrams of the variation of soil temperatures at selected places.

It is good to see that on each map the period to which the data refer is clearly stated; these periods vary in length however, from eight to fifty years. As with many atlases of this kind relatively few of the records used were completely acceptable throughout the stated period and consequently data for numerous stations have been adjusted by processes described. On this score it is doubtful whether the accuracy of the data was sufficient to warrant the very intricate patterns of isopleths on a few of the maps. A map showing the positions of observing stations would also have helped to indicate areas where the pattern is perhaps less reliable than elsewhere.

Despite the claim in the Preface, the completeness of this work is questionable. There are, for instance, no charts showing average monthly extremes or daily range of temperature, and data relating to wind frequencies, cloudiness and humidity are very limited. Clearly, however, a great deal of conscientious hard work has gone into this atlas and the over-all standard of production is good.

A.I.J.

Die Sommerniederschlage Mitteleuropas in den letzten 1½ Jahrhunderten und ihre Beziehungen zum Sonnenfleckenzyklus. By Franz Baur. Leipzig (Akademische Verlag. Geest & Portig), 1959. Pp. IV, 80, 2 Figures; 19 Tables. DM.9.

The introduction to this small book says that it was written firstly to provide a long series of homogeneous precipitation records for Central Europe and secondly to disprove a contention by H. Flohn that 1947 was the driest summer in 250 years. As a by-product, it was to supply a broader basis for earlier studies of solar-weather relationships.

Baur compiled rainfalls records from 10 stations, for June-July-August combined and for July-August only back to 1804. Gaps in the actual observation series are filled by correlation with single neighbouring stations. He computes the area mean rainfalls for all 10 stations and also for overlapping groups of stations representing northern, southern, western and eastern Central Europe. Finally, he lists the annual deviations from the long-term means for the whole area and for the four sub-regions. In order to show that his 10 stations are in fact representative, he investigated also a second group of 26 stations for which records from 1901 to 1956 had been available. The two series formed by the 10 and by the 26 stations agree reasonably well. From the complete analysis, it is deduced convincingly that 1911 was the driest Central European summer for 200 years, with 1904 running a close second – *pace* Flohn.

The remainder of the book deals with the synoptic character of dry or wet summers and with their relationship to the solar cycle. The author tries to prove that 3 maxima and 3 minima of rainfall tend to occur in each sun-spot cycle. A similar variation is said to occur in Edinburgh pressure records. A tentative explanation of the triple cycle is based on the assumption that emission from the photosphere and absorption of radiation in the solar atmosphere vary each separately with the sun-spot number. The author gives no physical reason for the variation in absorbtivity, but proves that if it did occur, the resulting variation in the total outgoing of radiation would have, in fact, several maxima per sun-spot period.

In a subsequent short discussion of solar-weather relationships, high-surface insolation in the tropics and strong heating of the ozone layer in high latitudes are said to be both conducive to the build-up of high pressures – and hence dryness at mid-latitudes. A statement that the mean solar constant for July 1955 was more than one per cent above the mean is not supported by any reference. A final chapter deals with the question why Central Europe suffered from 6 successive wet summers from 1953 to 1958. Baur places the blame on the very short interval between the sun-spot minimum in 1954 and the maximum in 1957.

The book deals only with summer rainfall in Central Europe. It is difficult to accept, without much reserve, any theory or hypothesis of solar-weather relationships which is based on such limited material. Conviction could only come from the establishment of world-wide effects, which fit into the framework of a coherent physical theory. The present book has the merit of providing some observational material and one or two suggestive ideas which might help in the search for a fuller picture.

E.B.K.

A handbook of Renaissance meteorology. By S. K. Heninger, Jr., Durham, North Carolina, (Duke University Press), and Cambridge University Press, 1960. Pp. xii, 269 : 5 Illustrations, 24 Headpieces. 60s.

About 350 B.C. Aristotle published his remarkable treatise *Meteorologica* in which he included in one system all those natural phenomena thought to occur in the atmosphere. This system was founded on a theory of exhalations due to the solar influence, and the various atmospheric phenomena, or meteors as he called them, were conceived as being due to these exhalations or to their mixing together in various proportions. The authority of this treatise was so great that no writer on the subject during the next 2,000 years did more than comment on the text with perhaps some added references to later examples of the phenomena. More than a hundred separate editions of the *Meteorologica*, with or without commentary, were published before the year 1600.

The author of the present book is concerned to show the impact of this venerable body of doctrine on the English writers of the Renaissance, with special reference to the imagery used by the greater poets of the Elizabethan age. He studies especially Spenser, Marlowe, Chapman, Jonson, Donne and Shakespeare. In a massive series of several hundred citations from the works of these poets and dramatists the author shows how deeply the conceptions of Aristotle are embedded in their poems and plays and in their use of meterological images.

This comprehensive study will be of great value to the student of English Renaissance literature who is also interested in pre-scientific meteorology. There are five appendices to the book, two of which contain useful bibliographies which could not be found elsewhere. The illustrations and the headpieces to the chapters are reproduced from printed books, mostly of the sixteenth century. The printing and the general format are excellent.

C.E.B.

Theoretical hydrodynamics. By L. M. Milne-Thomson. London (Macmillan & Co.), 4th Ed. 1960. Pp. xxv, 660; 4 Plates, Figures. Price 65s.

This well-known book was first published in 1938; the second and third editions appeared in 1949 and 1955 and now it has attained the dignity of a fourth edition. The present reviewer dealt with the second edition in the *Quarterly Journal* in 1950, and although the book has grown in the meantime, substantially it is the same. By far the greater part of the text deals with the dynamics of frictionless fluids and only one chapter, about 40 pages long, considers real (that is, viscous) fluids.

This book is, therefore, a dedicated account of the classical hydrodynamics, a study which reached its peak in the late nineteenth and early twentieth century. Its aim, as the author says, is ' to give a thorough, clear and methodical introductory exposition of the mathematical theory of fluid motion which will be useful in applications both to hydrodynamics and aerodynamics.' It differs from Lamb's great treatise in that it is written throughout in the language of vector analysis. The style is clear, the many figures are well drawn and reproduced and the mathematics is always elegant.

For those who wish to study classical hydrodynamics, and have the necessary facility in vector methods, this book is ideal. But to-day interest is centred more in the dynamics of real fluids and especially in turbulent motion. Professor Milne-Thomson still regards the applications of boundary-layer theory as ' of a tentative and empirical character ' and many important developments of recent years, such as the theory of buoyant jets, are not even mentioned. Turbulence is referred to only in two short sentences in the opening pages. Nor is there any mention of recent developments in atmospheric dynamics, although these provide admirable examples of the application of frictionless-fluid theory to real phenomena.

The book thus embodies a branch of mathematical physics which has given pure mathematicians much delight and has added greatly to our understanding of certain important matters in aerodynamics, such as the production of lift. In this limited field it is admirable.

It has little to say about the modern study of fluid motion, which has turned largely in the direction of statistical theory, boundary-layer theory and compressible flow.

O.G.S.

The world around us. Ed. by Sir Graham Sutton, London (The English Universities Press), 1st Ed., 1960. Pp. vi, 122; 58 Figures; 16s.

In his Presidential Address for the year 1960 Dr. J. M. Stagg drew the Society's attention to the necessity of extending its activities into neighbouring sciences. *The world around us* is concerned not only with conventional meteorology but also with those branches of geophysics which constitute ' neighbouring sciences.'

The book is, in fact, a series of essays based on the annual Christmas Lectures delivered at the Royal Institution in 1958 to predominantly youthful audiences. Although to this extent a popular book, its authoritative nature is clear from the list of contributors :

Dr. J. A. Ratcliffe on ' The ionosphere,' Dr. J. M. Stagg on ' The earth's magnetism,' Dr. R. L. F. Boyd on ' The exploration of the upper atmosphere,' Sir Graham Sutton on ' The lower atmosphere and its weather,' Dr. G. E. R. Deacon on ' The sea and its problems ' and Dr. G. de Q. Robin on ' The Antarctic.'

Yet these distinguished authors wear their learning lightly. Every essay is written in a direct and simple style and assumes just that knowledge of physics which a moderately intelligent schoolboy or schoolgirl can be expected to possess. Particular topics include the instrumentation of satellites, the nature of long oceanic waves and methods of research into sub-surface tides and curents. There is more about the great terrestrial dynamo described by Dr. Stagg in his Presidential Address. The last essay is mainly concerned with the climate and geophysical survey of Antarctica. (Surely Dr. Robin has oversimplified his theme here when he states by way of comparison that " in England . . . . in summer we have sixteen hours of sunshine a day."))

Unfortunately one feels that this book will fail to reach one of its most important targets : the intelligent but impecunious young. There is something illogical about an educational system, which, while crying out for young scientists, permits a book of this quality to sell for 16 shillings for 122 pages, while books of twice the length and little or no merit sell for less than one quarter of the price.

J.B.R.

Advances in geophysics. Vol. 6. Ed. by H. E. Landsberg and J. Van Miegham. *Proceedings of Symposium on Atmospheric Diffusion and Air Pollution* held at Oxford, 1958. Ed. by F. N. Frenkiel and P. A. Sheppard. Academic Press (London and New York), 1959. Pp. xvii, 470; Figs.; Tables. $12.00.

In 1956 the International Union of Theoretical and Applied Mechanics (I.U.T.A.M.) decided to invite the co-operation of the International Union of Geodesy and Geophysics (I.U.G.G.) in the organization of a Symposium on Atmospheric Diffusion and Air Pollution. Oxford was chosen as the location for the meeting to be held in the latter part of August 1958, and it fell to the joint secretaries, Dr. F. N. Frenkiel and Professor P. A. Sheppard, acting for an I.U.T.A.M.-I.U.G.G. Scientific Committee, to assemble an international group of eminent fluid dynamicists, meteorologists, oceanographers and other scientists for the presentation of papers on atmospheric diffusion and on the applications of diffusion research to the problems of air pollution.

This was indeed a most ambitiously planned Symposium and this volume is a testimony to the remarkable success which crowned the efforts of the organisers. Atmospheric diffusion is a highly specialized subject but it covers a wide range of problems and many lines of attack. The practical importance of the subject has increased greatly in recent years with the determination of communities to do away with the evils of urban pollution and also with the injection into the atmosphere of potentially dangerous radioactive substances by means of weapon tests and in the effluent from industrial nuclear reactors.

The meetings were divided into sessions : (1) A general survey, (2) recent findings on atmospheric turbulence, (3) theory of turbulent diffusion, (4) diffusion of heavy or finite particles, (5) transfer through the troposphere and stratosphere, (6) effects of thermal stratification on diffusion and (7) pollution patterns from point and area sources. At a final meeting the contributions and discussions of the seven sessions were reviewed, the various threads were drawn together so far as that was possible, and the way ahead was indicated in experimental work, in theoretical approaches and in technical application to everyday problems.

Altogether more than forty papers were presented at the Symposium and these have been printed in this volume together with notes on the discussions that took place. As well as the papers, the notes themselves must be awarded high marks for their crispness and point, achieved by the omission of the usual anecdotal preambles and courteous circumlocutions.

There is such a wealth of material in this volume that a reviewer might well sigh for the more spacious days such as when Macaulay criticized and expounded for many thousands of words. So many good papers are presented here that it would be difficult, and perhaps unwise, to select a few topics and call them highlights. The field covered is so comprehensive that this volume is indeed a review in itself and as such may be expected to remain a valuable source of reference for some considerable time. The production throughout is of the highest standard. The Editors are to be warmly congratulated on this culmination of their work.

P.J.M.

The Earth : its origin, history and physical constitution. By Sir Harold Jeffreys, Cambridge (University Press), 4th Ed., 1959. Pp. xvi, 420; 10 Plates, 30 Figures; Tables. 75s.

Comparison of the 2nd (1929), 3rd (1952), and 4th (1959) editions of this well-known text in geophysics reveals, as would be expected from the respective time intervals involved, that the changes made in this new edition are much less sweeping than those made in the 3rd edition. Various changes, additions and references to recent work have, however, been made in all sections of the new edition : they are most in evidence in the author's treatment of tidal friction and in the chapter relating to the thermal history of the earth.

Chapters 1 to 3 deal with theoretical and observational seismology, Chapters 4 to 8 with geodesy (figures of earth and moon, stress-differences in the earth, latitude variation, bodily tide, tidal friction). Chapters 9 to 11 deal, respectively, with the age of the earth, its thermal history, and the origin of its surface features.

Chapter 12 is devoted to ' special problems '– mainly rather brief discussion of a total of eleven miscellaneous topics which include periodicity of earthquakes, permanence of the continents, polar wandering, continental drift, surface features of the moon. In five appendices individual theoretical problems are dealt with.

As might be expected from the indications of the scope of the book contained in the extended title, the volume contains very little of direct interest to the meteorologist. While discussion of the various theories of climatic change which was contained in an appendix to some of the earlier editions is not featured in this volume, a point of substantial meteorological interest in the volume concerns, indirectly, this problem of climatic change. The question involved is that of the hypothesis of continental drift. The author remains, on physical grounds, a firm disbeliever in this hypothesis and is unmoved by the claims made by workers in palaeomagnetism to the effect that their results obtained in recent years lend strong and independent support to the hypothesis.

D.H.M.

Luftelektrizität und Radioaktivität. By H. Israel. Berlin, (Springer-Verlag), 1957. Pp. viii, 125; 86 Figures; 8 Tables. DM. 7.80.

This is an excellent and stimulating introduction to the electrical phenomena of the lower atmosphere and to the manner in which they depend on the physical properties of gaseous ions and atmospheric aerosol and on the properties of atmospheric turbulence. There is a useful brief chapter on atmospheric radioactivity which includes an account of the effect on atmosphere and earth of atomic explosions.

The author has attempted to compress an account of a very large field into a small book. To the reviewer the section on lightning and sferics, 10 pages in all, seems most obviously inadequate as a result of this compression.

On the whole it is a stimulating little volume for the reader who knows German or who wants to brush up his German.

T.W.W.

Lehrbuch der Luftelektrizität. By Erich von Kilinski. Leipzig (Akadem. Verlags. Geest und Portig K.-G.), 1958. Pp. viii, 141; 68 Figures; 18 Tables; DM. 17.

This is a well-written and attractively produced text-book of atmospheric electricity. It is up to date and gives a representative and well-balanced set of references. The general balance of the book however, is that which was traditional a generation ago, and it really is not possible, today, to give an adequate account of ' Shower and thundercloud electricity, lightning and atmospherics ' in a single chapter of 19 pages as is attempted. The rest of the book is very readable.

T.W.W.

NOTE : Readers of the reviews in the *Quarterly Journal* for April 1960 may wish to note that the books *Turbulent transfer in the lower atmosphere* by C. H. B. Priestley and *Radar meteorology* by L. J. Battan are published by the Cambridge University Press as well as by the University of Chicago Press as stated.

Obituary Notice

Mr. W. M. WITCHELL, B.Sc., F.R.A.S.

William Moody Witchell, who died on 6 December 1959 at the age of 80, was an astronomer by training and inclination, and a meteorologist by appointment. He entered the service of the Royal Observatory, Greenwich, as a Supernumerary Computer on 2 August 1894 and became an Established Computer on 29 January 1902 when the permanent staff was being enlarged. From 1906 to 1914 he held the important position of ' standard observer ' with the Airy transit circle – the instrument which had been internationally accepted since 1884 as defining the zero of terrestrial longitude. Witchell's status as standard observer of meridian transits meant that the personal equations of the other observers were reduced to his own; he may thus be said to have borne the prime responsibility for the derivation of Greenwich Mean Time over a period of nine years.

After the death of W. W. Bryant in January 1923 Witchell was appointed to succeed him as Superintendent of the Magnetic and Meteorological Department at the Royal Observatory. Although it brought him promotion to the grade of Assistant he did not find this change of occupation altogether congenial, for his main interests were always centred in the heavens rather than on the earth. Certainly he threw himself with characteristic energy into this new work, but he never developed the same enthusiasm for it as had been shown by his predecessor Bryant, who possessed an astonishing memory and was reputed to have by heart all the principal facts and figures about Greenwich weather back to 1841. Witchell remained at the head of ' Mag. and Met.' until his retirement on his 69th birthday, 26 April 1948. In that capacity he was responsible for the Magnetic Observatory at Abinger and assisted in the preparation of the world magnetic charts undertaken by the Observatory on behalf of the Admiralty's Hydrographic Department, to which, incidentally, he had been lent by the Astronomer Royal during the later stages of the 1914-18 war for work on submarine charts and salinities.

Mr. Witchell's record of about 53¾ years' unbroken service with the Royal Observatory, Greenwich, is unique in the long history of that institution. Throughout the entire period he was never once absent on sick leave. Parallels to this remarkable achievement must surely be very rare in any walk of life. As a young man, Witchell was a keen hockey player; for many years he was hon. secretary of the Royal Observatory Hockey Club. When he eventually found the game too strenuous for him he took to the road for exercise, often spending his holidays, even in advanced middle age, on tours by pedal bicycle to distant parts of the country.

Witchell's accession to the meteorological fraternity in 1923 proved to be a happy event for our Society, which he joined on 9 April 1924. He was elected to the Council in 1928. From 1930 to 1934 he served as one of the Hon. Secretaries and from 1937 to 1951 as Hon. Treasurer, displaying in both offices high competence and unflagging devotion to duty. On tendering his resignation from vice-presidency in June 1953 he mentioned that of some 230 Council meetings held since his first election as a councillor in 1928 he had been absent from only one, this solitary occasion being when he was bound for Athens on an expedition to observe a total eclipse of the sun. From 1924 until shortly before the records came to their lamented end in 1949 he contributed to this *Journal* the annual reports on the meteorological work at the Royal Observatory in continuation of the series published year by year back to 1871. The Society mourns the loss of a wise counsellor and an indefatigable worker for its welfare.

For some of the details of Mr. Witchell's career given above we are indebted to the courtesy of the Royal Astronomical Society.

E.L.H.

Corrigendum to January 1960 issue

R. J. TAYLOR—

On page 73, line 18, the definition of B should contain $T_{0\cdot5} - T_2$ in the numerator instead of $T_2 - T_{0\cdot5}$.

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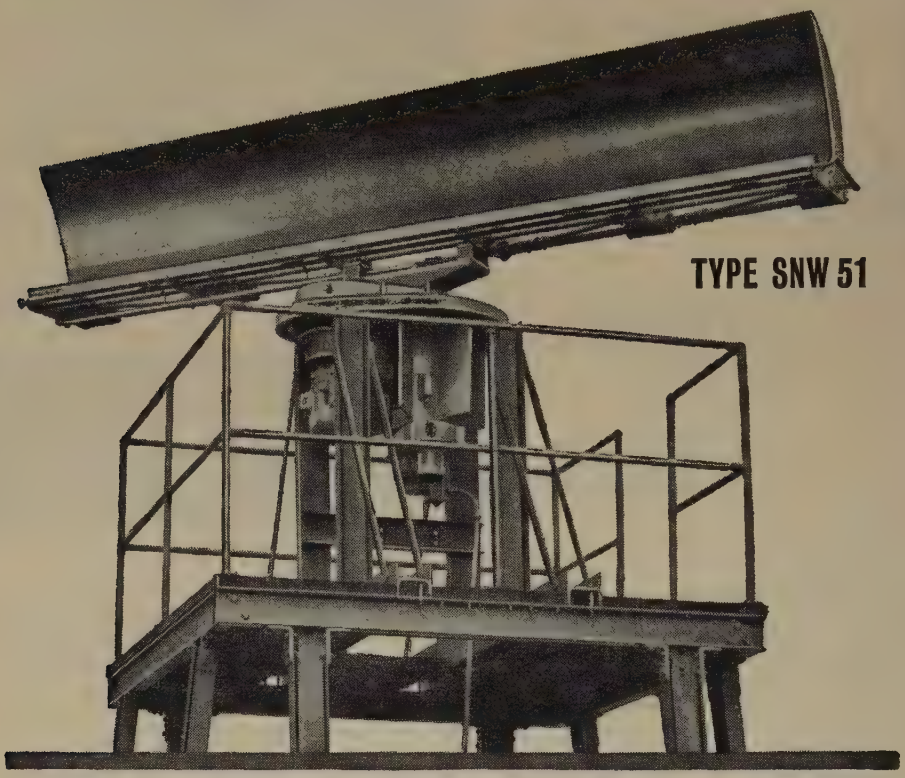

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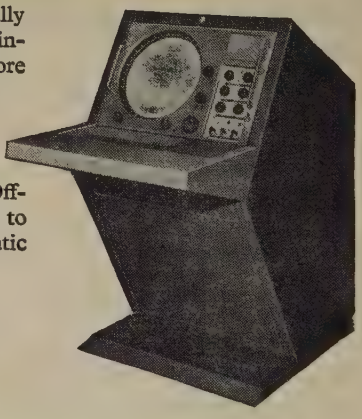

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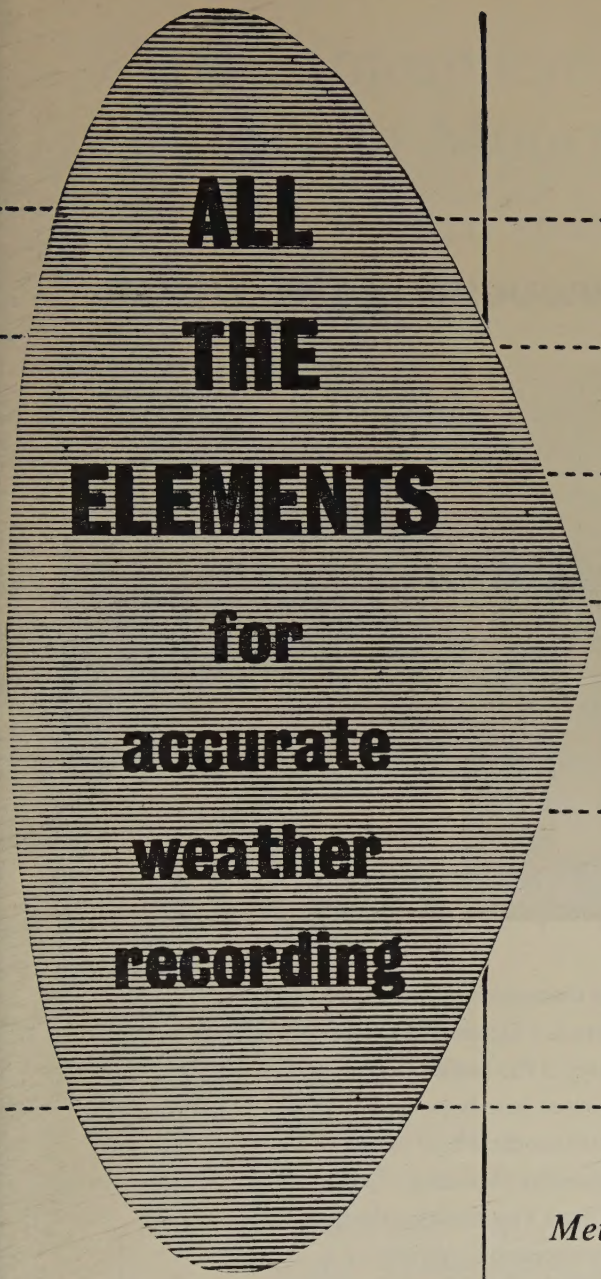

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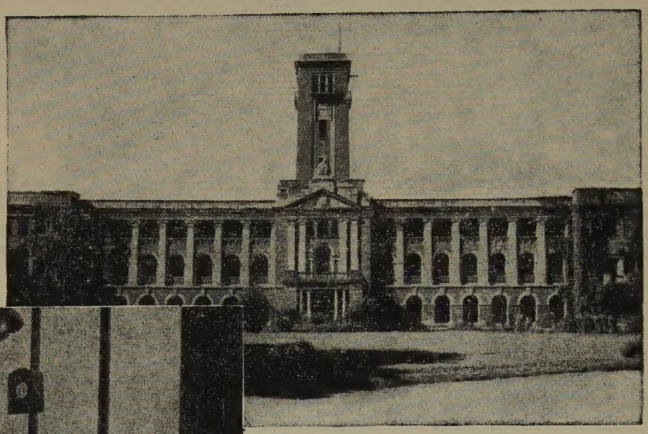

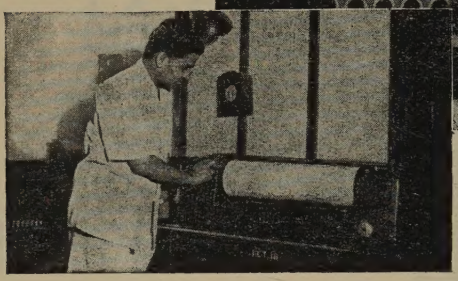

The Meteorological Office at Ganeshkhind Road, Poona, is under the direction of the Director General of Observatories, Shri S Basu. The Headquarters of the Indian Meteorological Society was based here between 1927 and 1942. Now the Weather Centre for the whole of the country is housed in this magnificent building.
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Announcement of third competition

The Royal Meteorological Society announces the third competition for the Napier Shaw Prize. The competition will be open to anyone without restriction of nationality. The prize of £100 is offered on this occasion for an original essay on '**The Upper Atmosphere.**'

> For the purpose of this competition the upper atmosphere is regarded as the atmosphere above the tropopause and below the exosphere. Essays on the dynamics, constitution or physics of the atmosphere in this region, or in any part of it, will be accepted, but an essay on purely electrical properties of the ionosphere is not sought. Preference will be given to essays relating two or more aspects of the state of the upper atmosphere. Essays on the inter-relations between the upper atmosphere and the troposphere will also be welcome.

The essay, which must be in English, may include material which the candidate has already published. Essays may be submitted jointly in the names of more than one author.

Candidates are required to submit three copies of their essay, which must be printed or typewritten. In the latter case, only one side of the paper should be used, double spacing should be employed and two of the copies may be good carbon copies. Unless the essay is printed, any mathematics should be written in carefully by hand. Although the scale of the essay will be considerably greater than that of an ordinary paper, candidates are recommended to consult, for general advice, the pamphlet, 'Preparation of papers for the Quarterly Journal.' This may be obtained, free of charge, on application to the Assistant Secretary.

It is required that any essay for which a prize is awarded shall be offered to the Society for publication.

All entries for the competition should be addressed to the Assistant Secretary, Royal Meteorological Society, 49 Cromwell Road, London, S.W.7 and marked clearly 'Napier Shaw Prize.'

Essays for the competition must be received at the Society's Offices *not later* than 1 January 1962.

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